1. Explain what you understand by
   (a) a population, (1)
   (b) a statistic. (1)

   A researcher took a sample of 100 voters from a certain town and asked them who they would vote for in an election. The proportion who said they would vote for Dr Smith was 35%.

   (c) State the population and the statistic in this case. (2)

   (d) Explain what you understand by the sampling distribution of this statistic. (1)

   (Total 5 marks)

2. A bag contains a large number of coins. It contains only 1p and 2p coins in the ratio 1:3

   (a) Find the mean $\mu$ and the variance $\sigma^2$ of the values of this population of coins. (3)

   A random sample of size 3 is taken from the bag.

   (b) List all the possible samples. (2)

   (c) Find the sampling distribution of the mean value of the samples. (6)

   (Total 11 marks)
3. A random sample $X_1, X_2, \ldots, X_n$ is taken from a population with unknown mean $\mu$ and unknown variance $\sigma^2$. A statistic $Y$ is based on this sample.

(a) Explain what you understand by the statistic $Y$. (2)

(b) Explain what you understand by the sampling distribution of $Y$. (1)

(c) State, giving a reason which of the following is not a statistic based on this sample.

\[
\begin{align*}
(i) & \quad \sum_{i=1}^{n} \frac{(X_i - \overline{X})^2}{n} \\
(ii) & \quad \sum_{i=1}^{n} \left( \frac{X_i - \mu}{\sigma} \right)^2 \\
(iii) & \quad \sum_{i=1}^{n} X_i^2
\end{align*}
\]

(2) (Total 5 marks)

4. (a) Explain what you understand by a census. (1)

Each cooker produced at GT Engineering is stamped with a unique serial number. GT Engineering produces cookers in batches of 2000. Before selling them, they test a random sample of 5 to see what electric current overload they will take before breaking down.

(b) Give one reason, other than to save time and cost, why a sample is taken rather than a census. (1)

(c) Suggest a suitable sampling frame from which to obtain this sample. (1)

(d) Identify the sampling units. (1) (Total 4 marks)
5. Before introducing a new rule the secretary of a golf club decided to find out how members might react to this rule.

(a) Explain why the secretary decided to take a random sample of club members rather than ask all the members.  

(b) Suggest a suitable sampling frame.  

(c) Identify the sampling units.  

(Total 3 marks)

6. A bag contains a large number of coins. Half of them are 1p coins, one third are 2p coins and the remainder are 5p coins.

(a) Find the mean and variance of the value of the coins.  

A random sample of 2 coins is chosen from the bag.

(b) List all the possible samples that can be drawn.  

(c) Find the sampling distribution of the mean value of these samples.  

(Total 13 marks)

7. Explain what you understand by

(a) a sampling unit,  

(b) a sampling frame,
8. (a) Explain what you understand by (i) a population and (ii) a sampling frame. (2)

The population and the sampling frame may not be the same.

(b) Explain why this might be the case. (1)

(c) Give an example, justifying your choices, to illustrate when you might use
   (i) a census,
   (ii) a sample. (4)

(Total 7 marks)

9. Explain briefly what you understand by
   (a) a sampling frame, (1)

   (b) a statistic. (2)

(Total 3 marks)

10. A large dental practice wishes to investigate the level of satisfaction of its patients.
    (a) Suggest a suitable sampling frame for the investigation. (1)
(b) Identify the sampling units. (1)

(c) State one advantage and one disadvantage of using a sample survey rather than a census. (2)

(d) Suggest a problem that might arise with the sampling frame when selecting patients. (1)

(Total 5 marks)
11. A magazine has a large number of subscribers who each pay a membership fee that is due on January 1st each year. Not all subscribers pay their fee by the due date. Based on correspondence from the subscribers, the editor of the magazine believes that 40% of subscribers wish to change the name of the magazine. Before making this change the editor decides to carry out a sample survey to obtain the opinions of the subscribers. He uses only those members who have paid their fee on time.

(a) Define the population associated with the magazine.  

(b) Suggest a suitable sampling frame for the survey.  

(c) Identify the sampling units.  

(d) Give one advantage and one disadvantage that would have resulted from the editor using a census rather than a sample survey.  

As a pilot study the editor took a random sample of 25 subscribers.

(e) Assuming that the editor’s belief is correct, find the probability that exactly 10 of these subscribers agreed with changing the name.  

In fact only 6 subscribers agreed to the name being changed.

(f) Stating your hypotheses clearly test, at the 5% level of significance, whether or not the percentage agreeing to the change is less that the editor believes.  

The full survey is to be carried out using 200 randomly chosen subscribers.

(g) Again assuming the editor’s belief to be correct and using a suitable approximation, find the probability that in this sample there will be least 71 but fewer than 83 subscribers who agree to the name being changed.

(Total 20 marks)
12. An athletics teacher has kept careful records over the past 20 years of results from school sports days. There are always 10 competitors in the javelin competition. Each competitor is allowed 3 attempts and the teacher has a record of the distances thrown by each competitor at each attempt. The random variable $D$ represents the greatest distance thrown by each competitor and the random variable $A$ represents the number of the attempt in which the competitor achieved their greatest distance.

(a) State which of the two random variables $D$ or $A$ is continuous. (1)

A new athletics coach wishes to take a random sample of the records of 36 javelin competitors.

(b) Specify a suitable sampling frame and explain how such a sample could be taken. (2)

The coach assumes that $P(A = 2) = \frac{1}{3}$, and is therefore surprised to find that 20 of the 36 competitors in the sample achieved their greatest distance on their second attempt.

Using a suitable approximation, and assuming that $P(A = 2) = \frac{1}{3}$,

(c) find the probability that at least 20 of the competitors achieved their greatest distance on their second attempt. (6)

(d) Comment on the assumption that $P(A = 2) = \frac{1}{3}$. (2)

(Total 11 marks)

13. A company has a large number of regular users logging onto its website. On average 4 users every hour fail to connect to the company’s website at their first attempt.

(a) Explain why the Poisson distribution may be a suitable model in this case. (1)

Find the probability that, in a randomly chosen 2 hour period,

(b) (i) all users connect at their first attempt,

(ii) at least 4 users fail to connect at their first attempt. (5)
The company suffered from a virus infecting its computer system. During this infection it was found that the number of users failing to connect at their first attempt, over a 12 hour period, was 60.

(c) Using a suitable approximation, test whether or not the mean number of users per hour who failed to connect at their first attempt had increased. Use a 5% level of significance and state your hypotheses clearly.

(9)

(Total 15 marks)

14. A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample.

(2)

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is \( \frac{1}{4} \). The probability of rejection in either tail should be as close as possible to 0.025

(3)

(c) Find the actual significance level of this test.

(2)

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company’s claim in the light of this value. Justify your answer.

(2)

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly.

(6)

(Total 15 marks)
15. (a) Define the critical region of a test statistic.

A discrete random variable \( x \) has a Binomial distribution \( B(30, p) \). A single observation is used to test \( H_0 : p = 0.3 \) against \( H_1 : p \neq 0.3 \)

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each tail which should be as close as possible to 0.005

(c) Write down the actual significance level of the test.

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region.

(Total 10 marks)

16. An effect of a certain disease is that a small number of the red blood cells are deformed. Emily has this disease and the deformed blood cells occur randomly at a rate of 2.5 per ml of her blood. Following a course of treatment, a random sample of 2 ml of Emily’s blood is found to contain only 1 deformed red blood cell.

Stating your hypotheses clearly and using a 5% level of significance, test whether or not there has been a decrease in the number of deformed red blood cells in Emily’s blood.

(Total 6 marks)

17. Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager questions whether or not there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(a) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager’s question. You should state the probability of rejection in each tail which should be less than 0.05.
18. A single observation $x$ is to be taken from a Binomial distribution $B(20, p)$.
This observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(a) Using a 5% level of significance, find the critical region for this test. The probability of rejecting either tail should be as close as possible to 2.5%.

(b) State the actual significance level of this test.

(c) State a conclusion that can be drawn based on this value giving a reason for your answer.

(Total 7 marks)

19. A web server is visited on weekdays, at a rate of 7 visits per minute. In a random one minute on a Saturday the web server is visited 10 times.

(a) (i) Test, at the 10% level of significance, whether or not there is evidence that the rate of visits is greater on a Saturday than on weekdays. State your hypotheses clearly.

(ii) State the minimum number of visits required to obtain a significant result.

(Total 7 marks)
(b) State an assumption that has been made about the visits to the server. (1)

In a random two minute period on a Saturday the web server is visited 20 times.

(c) Using a suitable approximation, test at the 10% level of significance, whether or not the rate of visits is greater on a Saturday. (6)

(Total 14 marks)

20. A test statistic has a Poisson distribution with parameter \( \lambda \).

Given that

\[ H_0 : \lambda = 9, \quad H_1 : \lambda \neq 9 \]

(a) find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%. (3)

(b) State the probability of incorrectly rejecting \( H_0 \) using this critical region. (2)

(Total 5 marks)

21. Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm. She decides to try a new fertiliser. In a random sample of 40 ripe tomatoes, 18 have a diameter greater than 4 cm. Dhriti claims that the new fertiliser has increased the probability of a ripe tomato being greater than 4 cm in diameter.

Test Dhriti’s claim at the 5% level of significance. State your hypotheses clearly. (Total 7 marks)

22. (a) Explain what you understand by

(i) a hypothesis test,

(ii) a critical region. (3)
During term time, incoming calls to a school are thought to occur at a rate of 0.45 per minute. To test this, the number of calls during a random 20 minute interval, is recorded.

(b) Find the critical region for a two-tailed test of the hypothesis that the number of incoming calls occurs at a rate of 0.45 per 1 minute interval. The probability in each tail should be as close to 2.5% as possible.

(c) Write down the actual significance level of the above test.

In the school holidays, 1 call occurs in a 10 minute interval.

(d) Test, at the 5% level of significance, whether or not there is evidence that the rate of incoming calls is less during the school holidays than in term time.

23. Bacteria are randomly distributed in a river at a rate of 5 per litre of water. A new factory opens and a scientist claims it is polluting the river with bacteria. He takes a sample of 0.5 litres of water from the river near the factory and finds that it contains 7 bacteria. Stating your hypotheses clearly test, at the 5% level of significance, the claim of the scientist.

24. Linda regularly takes a taxi to work five times a week. Over a long period of time she finds the taxi is late once a week. The taxi firm changes her driver and Linda thinks the taxi is late more often. In the first week, with the new driver, the taxi is late 3 times.

You may assume that the number of times a taxi is late in a week has a Binomial distribution.

Test, at the 5% level of significance, whether or not there is evidence of an increase in the proportion of times the taxi is late. State your hypotheses clearly.
25. Past records from a large supermarket show that 20% of people who buy chocolate bars buy the family size bar. On one particular day a random sample of 30 people was taken from those that had bought chocolate bars and 2 of them were found to have bought a family size bar.

(a) Test at the 5% significance level, whether or not the proportion $p$, of people who bought a family size bar of chocolate that day had decreased. State your hypotheses clearly.

The manager of the supermarket thinks that the probability of a person buying a gigantic chocolate bar is only 0.02. To test whether this hypothesis is true the manager decides to take a random sample of 200 people who bought chocolate bars.

(b) Find the critical region that would enable the manager to test whether or not there is evidence that the probability is different from 0.02. The probability of each tail should be as close to 2.5% as possible.

(c) Write down the significance level of this test.

(Total 13 marks)

26. Breakdowns occur on a particular machine at random at a mean rate of 1.25 per week.

(a) Find the probability that fewer than 3 breakdowns occurred in a randomly chosen week.

Over a 4 week period the machine was monitored. During this time there were 11 breakdowns.

(b) Test, at the 5% level of significance, whether or not there is evidence that the rate of breakdowns has changed over this period. State your hypotheses clearly.

(Total 11 marks)

27. It is known from past records that 1 in 5 bowls produced in a pottery have minor defects. To monitor production a random sample of 25 bowls was taken and the number of such bowls with defects was recorded.
(a) Using a 5% level of significance, find critical regions for a two-tailed test of the hypothesis that 1 in 5 bowls have defects. The probability of rejecting, in either tail, should be as close to 2.5% as possible. (6)

(b) State the actual significance level of the above test. (1)

At a later date, a random sample of 20 bowls was taken and 2 of them were found to have defects.

(c) Test, at the 10% level of significance, whether or not there is evidence that the proportion of bowls with defects has decreased. State your hypotheses clearly. (7)

28. A teacher thinks that 20% of the pupils in a school read the Deano comic regularly.

He chooses 20 pupils at random and finds 9 of them read Deano.

(a) (i) Test, at the 5% level of significance, whether or not there is evidence that the percentage of pupils that read Deano is different from 20%. State your hypotheses clearly.

(ii) State all the possible numbers of pupils that read Deano from a sample of size 20 that will make the test in part (a)(i) significant at the 5% level. (9)

The teacher takes another 4 random samples of size 20 and they contain 1, 3, 1 and 4 pupils that read Deano.

(b) By combining all 5 samples and using a suitable approximation test, at the 5% level of significance, whether or not this provides evidence that the percentage of pupils in the school that read Deano is different from 20%. (8)

(c) Comment on your results for the tests in part (a) and part (b). (2)

(Total 14 marks)
29. In an experiment, there are 250 trials and each trial results in a success or a failure.
(a) Write down two other conditions needed to make this into a binomial experiment.

It is claimed that 10% of students can tell the difference between two brands of baked beans. In a random sample of 250 students, 40 of them were able to distinguish the difference between the two brands.

(b) Using a normal approximation, test at the 1% level of significance whether or not the claim is justified. Use a one-tailed test.

(c) Comment on the acceptability of the assumptions you needed to carry out the test.

(Total 10 marks)

30. Over a long period of time, accidents happened on a stretch of road at random at a rate of 3 per month.

Find the probability that
(a) in a randomly chosen month, more than 4 accidents occurred,

(b) in a three-month period, more than 4 accidents occurred.

At a later date, a speed restriction was introduced on this stretch of road. During a randomly chosen month only one accident occurred.

(c) Test, at the 5% level of significance, whether or not there is evidence to support the claim that this speed restriction reduced the mean number of road accidents occurring per month.

(Total 10 marks)
The speed restriction was kept on this road. Over a two-year period, 55 accidents occurred.

(d) Test, at the 5\% level of significance, whether or not there is now evidence that this speed restriction reduced the mean number of road accidents occurring per month.

(Total 16 marks)

31. Brad planted 25 seeds in his greenhouse. He has read in a gardening book that the probability of one of these seeds germinating is 0.25. Ten of Brad’s seeds germinated. He claimed that the gardening book had underestimated this probability. Test, at the 5\% level of significance, Brad’s claim. State your hypotheses clearly.

(Total 7 marks)

32. (a) Explain what you understand by a critical region of a test statistic.

(b) The number of breakdowns per day in a large fleet of hire cars has a Poisson distribution with mean \( \frac{1}{7} \).

(b) Find the probability that on a particular day there are fewer than 2 breakdowns.

(c) Find the probability that during a 14-day period there are at most 4 breakdowns.

The cars are maintained at a garage. The garage introduced a weekly check to try to decrease the number of cars that break down. In a randomly selected 28-day period after the checks are introduced, only 1 hire car broke down.

(d) Test, at the 5\% level of significance, whether or not the mean number of breakdowns has decreased. State your hypotheses clearly.

(Total 15 marks)
33. Vehicles pass a particular point on a road at a rate of 51 vehicles per hour.
   (a) Give two reasons to support the use of the Poisson distribution as a suitable model for the number of vehicles passing this point. 
      Find the probability that in any randomly selected 10 minute interval
   (b) exactly 6 cars pass this point,
   (c) at least 9 cars pass this point.
   After the introduction of a roundabout some distance away from this point it is suggested that the number of vehicles passing it has decreased. During a randomly selected 10 minute interval 4 vehicles pass the point.
   (d) Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that the number of vehicles has decreased. State your hypotheses clearly.

(Total 13 marks)

34. From past records a manufacturer of ceramic plant pots knows that 20% of them will have defects. To monitor the production process, a random sample of 25 pots is checked each day and the number of pots with defects is recorded.
   (a) Find the critical regions for a two-tailed test of the hypothesis that the probability that a plant pot has defects is 0.20. The probability of rejection in either tail should be as close as possible to 2.5%.
   (b) Write down the significance level of the above test.

(Total 13 marks)
A garden centre sells these plant pots at a rate of 10 per week. In an attempt to increase sales, the price was reduced over a six-week period. During this period a total of 74 pots was sold.

(c) Using a 5% level of significance, test whether or not there is evidence that the rate of sales per week has increased during this six-week period.

(Total 13 marks)

35. A single observation \( x \) is to be taken from a Poisson distribution with parameter \( \lambda \). This observation is to be used to test \( H_0 : \lambda = 7 \) against \( H_1 : \lambda \neq 7 \).

(a) Using a 5% significance level, find the critical region for this test assuming that the probability of rejecting in either tail is as close as possible to 2.5%.

(b) Write down the significance level of this test.

(c) State a conclusion that can be drawn based on this value.

(Total 8 marks)

36. From past records a manufacturer of glass vases knows that 15% of the production have slight defects. To monitor the production, a random sample of 20 vases is checked each day and the number of vases with slight defects is recorded.

(a) Using a 5% significance level, find the critical regions for a two-tailed test of the hypothesis that the probability of a vase with slight defects is 0.15. The probability of rejecting, in either tail, should be as close as possible to 2.5%.

(b) State the actual significance level of the test described in part (a).

A shop sells these vases at a rate of 2.5 per week. In the 4 weeks of December the shop sold 15 vases.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence that the rate of sales per week had increased in December.

(Total 12 marks)

MARK SCHEME

1. (a) A population is collection of all items  

   B1 1

   Note

   B1 – collection/group all items – need to have /imply all eg entire/complete/every

   (b) (A random variable) that is a function of the sample which contains  

   B1 1
no unknown quantities/parameters.

**Note**

**B1** – needs function/calculation(o.e.) of the sample/random variables/observations and no unknown quantities/parameters(o.e.)

NB do not allow unknown variables
e.g. “A calculation based solely on observations from a given sample.” B1
“A calculation based only on known data from a sample” B1
“A calculation based on known observations from a sample” B0

Solely/only imply no unknown quantities

(c) The voters in the town

Percentage/proportion voting for Dr Smith [2]

**Note**

**B1** – Voters

Do not allow 100 voters.

**B1** – percentage/ proportion voting (for Dr Smith)
the **number** of people voting (for Dr Smith)
Allow 35% of people voting (for Dr Smith)
Allow 35 people voting (for Dr Smith)
Do not allow 35% or 35 alone

(d) Probability Distribution of those voting for Dr Smith from all possible samples (of size 100)

**Note**

**B1** – answers must include all three of these features
(i) All possible samples,
(ii) their associated probabilities,
(iii) context of voting for Dr Smith.

e.g “It is all possible values of the percentage and their associated probabilities.” B0 no context
2. (a)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$1p$</th>
<th>$2p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{3}{4}$</td>
</tr>
</tbody>
</table>

$\mu = 1 \times \frac{1}{4} + 2 \times \frac{3}{4} = \frac{7}{4}$ or $\frac{3}{4}$ or 1.75 \hspace{1cm} \text{B1}

$\sigma^2 = 1^2 \times \frac{1}{4} + 2^2 \times \frac{3}{4} - \left( \frac{7}{4} \right)^2 \hspace{1cm} \text{M1}$

$= \frac{3}{16}$ or 0.1875 \hspace{1cm} \text{A1 3}

\textbf{Note}

B1 1.75 oe

M1 for using $\sum (x^2 p) - \mu^2$

A1 0.1875 oe

(b) (1,1,1), (1,1,2) any order, (1,2,2) any order, (2,2,2)

(1,2,1) (2,1,1) (2,1,2) (2,2,1)

all 8 cases considered. \hspace{1cm} \text{B1 2}

May be implied by 3 * (1,1,2) and 3 * (1,2,2)

\textbf{Note}

ignore repeats

(c)

$\bar{x}$ \hspace{1cm} 1 \hspace{1cm} $\frac{4}{3}$ \hspace{1cm} $\frac{5}{3}$ \hspace{1cm} 2

$P(\bar{X} = \bar{x}) \hspace{1cm} \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64} \hspace{1cm} 3 \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{9}{64} \hspace{1cm} 3 \times \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64} \hspace{1cm} \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$

B1 M1 A1 M1 A1A1 6
Note
1\textsuperscript{st} B1 4 correct means (allow repeats)
1\textsuperscript{st} M1 for $p^3$ for either of the ends
1\textsuperscript{st} A1 for 1/64 or awrt 0.016 and 27/64 or awrt 0.422
2\textsuperscript{nd} M1 $3 \times p^2(1-p)$ for either of the middle two
0 < p < 1
May be awarded for finding the probability of the
3 samples with mean of either 4/3 or 5/3 .
2\textsuperscript{nd} A1 for 9/64 (or 3/64 three times) and 27/64
(or 9/64 three times) accept awrt 3dp.
3\textsuperscript{rd} A1 fully correct table, accept awrt 3dp.

3. (a) A statistic is a function of $X_1, X_2, \ldots, X_n$ that does not contain any unknown parameters

Note
Examples of other acceptable wording:
B1 e.g. is a function of the sample or the data / is a quantity calculated from the sample or the data / is a random variable calculated from the sample or the data
B1 e.g. does not contain any unknown parameters/quantities contains only known parameters/quantities only contains values of the sample
Y is a function of $X_1, X_2, \ldots, X_n$ that does not contain any unknown parameters is a function of the values of a sample with no unknowns is a function of the sample values is a function of all the data values A random variable calculated from the sample A random variable consisting of any function A function of a value of the sample A function of the sample which contains no other values/ parameters
(b) The probability distribution of \( Y \) or the distribution of all possible values of \( Y \) (o.e.)

**Note**

Examples of other acceptable wording

All possible values of the statistic together with their associated probabilities

(c) Identify (ii) as not a statistic

Since it contains unknown parameters \( \mu \) and \( \sigma \).

**Note**

1\(^{\text{st}}\) B1 for selecting only (ii)

2\(^{\text{nd}}\) B1 for a reason. This is dependent upon the first B1. Need to mention at least one of \( \mu \) (mean) or \( \sigma \) (standard deviation or variance) or unknown parameters.

Examples

since it contains \( \mu \) B1

since it contains \( \sigma \) B1

since it contains unknown parameters/quantities B1

since it contains unknowns B0

4. (a) A census is when every member of the population is investigated.

B1 Need one word from each group

(1) Every member /all items / entire /oe

(2) population/collection of individuals/sampling frame/oe

enumerating the population on its own gets B0

(b) There would be no cookers left to sell.

B1 Idea of Tests to destruction. Do not accept cheap or quick

(c) A list of the unique identification numbers of the cookers.

B1 Idea of list/register/database of cookers/serial numbers
(d) A cooker

**B1** cooker(s) / serial number(s)

The sample of 5 cookers or every 400th cooker gets B1

5. (a) Saves time / cheaper / easier

*any one*

or

A census / asking all members takes a long time or is expensive or difficult to carry out

(b) List, register or database of all club members / golfers

or

Full membership list

(c) Club member(s)

6. (a)

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X = x)</td>
<td>1/2</td>
<td>1/3</td>
<td>1/6</td>
</tr>
</tbody>
</table>

Mean = 1 \times \frac{1}{2} + 2 \times \frac{1}{3} + 5 \times \frac{1}{6} = 2 or 0.02 \sum x \cdot p(x) need \frac{1}{2} and \sqrt[3]{M1 A1}

Variance = 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{3} + 5^2 \times \frac{1}{6} - 2^2 = 2 or 0.0002 M A1 4

(b) \sum x^2 \cdot p(x) - \lambda^2

(1,1) \hspace{1cm} B2
(1,2) and (2,1)
(1,5) and (5,1) LHS -1 B1 3

\text{e.e.}

(2,2)
(2,5) and (5,2) repeat of “theirs” on RHS B1
(5,5)
(c)
\[
\begin{align*}
\bar{x} & \quad 1 & 1.5 & 2 & 3 & 3.5 & 5 \\
P(\bar{X} = \bar{x}) & \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} & \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} & \frac{1}{6} & 2 \times \frac{1}{3} \times \frac{1}{6} = \frac{1}{9} & \frac{1}{36} \\
\end{align*}
\]

\[\frac{1}{4} \text{ M1A1}
\]
\[1.5+, \text{lee} \quad \text{M1 M2} \quad 6\]

7. (a) Individual member or element of the population or sampling frame \hspace{1cm} B1 \hspace{1cm} 1

(b) A list of all sampling units or all the population \hspace{1cm} B1 \hspace{1cm} 1

(c) All possible samples are chosen from a population; the values of a statistic and the associated probabilities is a sampling distribution \hspace{1cm} B1 B1 \hspace{1cm} 2

8. (a) (i) A collection of individuals or items \hspace{1cm} B1

(ii) A list of all sampling units in the population \hspace{1cm} B1 \hspace{1cm} 2

(b) Not always possible to keep this list up to date \hspace{1cm} B1 \hspace{1cm} 1

(c) (i) eg:- Pupils in year 12 – small easily listed population \hspace{1cm} B1

Population known & easily accessed \hspace{1cm} B1

(ii) Students in a University – large not easily listed population \hspace{1cm} B1

Population known but too time consuming/expensive to interview all of them \hspace{1cm} B1 \hspace{1cm} 4

OR

(c) (i) Definition of census by example \hspace{1cm} B1

(ii) Definition of sample by example \hspace{1cm} B1

9. (a) A list of (all) the members of the population \hspace{1cm} B1 \hspace{1cm} 1

A random variable that is a function of a random sample that contains no unknown parameters \hspace{1cm} B1 \hspace{1cm} 2

[3]
10. (a) List of patients registered with the practice. 
   Require ‘list’ or ‘register’ or database or similar  
   B1  1

(b) The patient(s)  
   B1  1

(c) Adv: Quicker, cheaper, easier, used when testing results in destruction of item, quality of info about each sampling unit is often better. Any one  
   Disadv: Uncertainty due to natural variation, uncertainty due to bias, possible bias as sampling frame incomplete, bias due to subjective choice of sample, bias due to non-response Any one  
   B1  2

(d) Non-response due to patients registered with the practice but who have left the area  
   B1  1

[5]

11. (a) All subscribers to the magazine  
   B1  1

(b) A list of all members that had paid their subscriptions  
   B1  1

(c) Members who have paid  
   B1  1

(d) Advantage: total accuracy  
   Disadvantage: time consuming to obtain data and analyse it  
   B1  2

(e) Let \( X \) represent the number agreeing to change the name  
   \( \therefore X \sim B(25, 0.4) \)  
   \[ P(X = 10) = P(X \leq 10) - P(X \leq 9) = 0.1612 \]  
   M1  A1  3

(f) \( H_0: p = 0.40, H_1: p < 0.40 \)  
   \[ P(X \leq 6) = 0.0736 > 0.05 \Rightarrow \text{not significant} \]  
   M1  A1  5

   No reason to reject \( H_0 \) and conclude % is less than the editor believes  
   A1  5

(g) Let \( X \) represent the number agreeing to change the name  
   \( \therefore X \sim B(200, 0.4) \)  
   \[ P(71 \leq X < 83) \approx P(70.5 \leq Y < 82.5) \text{ where } Y \sim N(80, 48) \]  
   \[ \approx P\left( \frac{70.5 - 80}{\sqrt{48}} \leq X < \frac{82.5 - 80}{\sqrt{48}} \right) \]  
   M1  M1  7

   \[ \approx P(-1.37 \leq X < 0.36) \]  
   A1  A1  7

   \[ = 0.5533 \]  
   A1  7

[20]
12. (a) \( D \) is continuous \hspace{1cm} \text{B1 1}

(b) Sampling Frame is the list of competitors or their results, \hspace{1cm} \text{B1}
\[ \text{e.g. label the results 1—200 and randomly select 36 of them} \hspace{1cm} \text{B1 2} \]

(c) \( X = \) no. of competitors with \( A = 2 \) \hspace{1cm} \( X \sim B(36, \frac{1}{3}) \)
\[ X \approx N(12, 8) \hspace{1cm} \text{M1 A1} \]
\[ P(X \geq 20) \approx P \left( Z \geq \frac{19.5 - 12}{\sqrt{8}} \right) \pm \frac{1}{2}, \text{‘z’} \hspace{1cm} \text{M1 M1} \]
\[ = P(Z \geq 2.65...) \hspace{1cm} \text{A1} \]
\[ = 1 - 0.9960 = 0.004 \hspace{1cm} \text{A1 6} \]

(d) Probability is very low, so assumption of \( P(A = 2) = \frac{1}{3} \) is unlikely. \hspace{1cm} \text{B1 B1 2}
(Suggests \( P(A = 2) \) might be higher.)

[11]

13. (a) Connecting occurs at random/independently, singly or at a constant rate \hspace{1cm} \text{B1 1}

\textbf{Note}
\[ \text{B1 Any one of randomly/independently/singly/constant rate. Must} \]
\[ \text{have context of connection/logging on/fail} \]

(b) \( \text{Po (8)} \hspace{1cm} \text{B1} \)

\textbf{Note}
\[ \text{B1 Writing or using Po(8) in (i) or (ii)} \]

(i) \( P(X = 0) = 0.0003 \hspace{1cm} \text{M1 A1} \)

\textbf{Note}
\[ \text{M1 for writing or finding } P(X = 0) \]
\[ \text{A1 awrt 0.0003} \]

(ii) \[ P(X \geq 4) = 1 - P(X \leq 3) \hspace{1cm} \text{M1} \]
\[ = 1 - 0.0424 \hspace{1cm} \text{A1 5} \]
\[ = 0.9576 \]

\textbf{Note}
\[ \text{M1 for writing or finding } 1 - P(X \leq 3) \]
\[ \text{A1 awrt 0.958} \]
(c) \( H_0 : \lambda = 4 \, (48) \quad H_1 : \lambda > 4 \, (48) \)

Method 1

\[
P(X \geq 59.5) = \frac{P \left( Z \geq \frac{59.5 - 48}{\sqrt{48}} \right)}{48} = 1.6449
\]

Method 2

\[
P(Z \geq 1.66) = 1 \times 0.9515 = 0.0485
\]

\[0.0485 < 0.05\]

Reject \( H_0 \). Significant. 60 lies in the Critical region

The number of failed connections at the first attempt has increased.

---

**Note**

**B1** both hypotheses correct. Must use \( \lambda \) or \( \mu \)

**M1** identifying normal

**A1** using or seeing mean and variance of 48

These first two marks may be given if the following are seen in the standardisation formula: 48 and \( \sqrt{48} \) or awrt 6.93

**M1** for attempting a continuity correction (Method 1: 60 ± 0.5 / Method 2: \( x \pm 0.5 \))

**M1** for standardising using their mean and their standard deviation and using either Method 1 \([59.5, 60 or 60.5. \, accept \pm z \, value]\) Method 2 \([x \pm 0.5 \, and \, equal \, to \, a \, \pm z \, value]\)

**A1** correct \( z \) value awrt ±1.66 or ± \( \frac{59.5 - 48}{\sqrt{48}} \), or \( \frac{x - 0.5 - 48}{\sqrt{48}} = 1.6449 \)

**A1** awrt 3 sig fig in range 0.0484 – 0.0485, awrt 59.9

**M1** for “reject \( H_0 \)” or “significant” maybe implied by “correct contextual comment”

If one tail hypotheses given follow through “their prob” and \( 0.05, p < 0.5 \)

If two tail hypotheses given follow through “their prob” with \( 0.025, p < 0.5 \)

If one tail hypotheses given follow through “their prob” and \( 0.95, p > 0.5 \)

If two tail hypotheses given follow through “their prob” with \( 0.975, p > 0.5 \)

If no \( H_1 \) given they get M0

**A1** ft correct contextual statement followed through from their prob and \( H_1 \), need the words number of failed connections/log ons has increased o.e.

Allow “there are more failed connections”

NB A correct contextual statement **alone** followed through from their prob and \( H_1 \) gets M1 A1
14. (a) 2 outcomes/faulty or not faulty/success or fail  
A constant probability  
Independence  
Fixed number of trials (fixed n)  

**Note**  
B1 B1 one mark for each of any of the four statements. Give first B1 if only one correct statement given. No context needed.

(b) \(X \sim B(50, 0.25)\)  
P(\(X \leq 6\)) = 0.0194  
P(\(X \leq 7\)) = 0.0453  
P(\(X \geq 18\)) = 0.0551  
P(\(X \geq 19\)) = 0.0287  
CR \(X \leq 6\) and \(X \geq 19\)  

**Note**  
M1 for writing or using \(B(50, 0.25)\) also may be implied by both CR being correct. Condone use of \(P\) in critical region for the method mark.  
A1 \((X) \leq 6\) o.e. [0, 6] DO NOT accept P(\(X \leq 6\))  
A1 \((X) \geq 19\) o.e. [19, 50] DO NOT accept P(\(X \geq 19\))

(c) 0.0194 + 0.0287 = 0.0481  

**Note**  
M1 Adding two probabilities for two tails. Both probabilities must be less than 0.5  
A1 awrt 0.0481

(d) 8(It) is not in the Critical region or 8(It) is not significant  
or 0.0916 > 0.025;  
There is evidence that the probability of a faulty bolt is 0.25  
or the company’s claim is correct  

**Note**  
M1 one of the given statements followed through from their CR.  
A1 contextual comment followed through from their CR.  
NB A correct contextual comment alone followed through from their CR will get M1 A1
(e) \( H_0 : p = 0.25 \quad H_1 : p < 0.25 \)

\[ P(X \leq 5) = 0.0070 \quad \text{or} \quad \text{CR} \ X \leq 5 \]

0.007 < 0.01;

5 is in the critical region, reject \( H_0 \), significant.

There is evidence that the probability of faulty bolts has decreased.

Note

B1 for \( H_0 \) must use \( p \) or \( \pi \) (pi)

B1 for \( H_1 \) must use \( p \) or \( \pi \) (pi)

M1 for finding or writing \( P(X \leq 5) \) or attempting to find a critical region or a correct critical region

A1 wrt 0.007/CR \( X \leq 5 \)

M1 correct statement using their Probability and 0.01 if one tail test

or a correct statement using their Probability and 0.005 if two tail test.

The 0.01 or 0.005 needn’t be explicitly seen but implied by correct statement compatible with their \( H_1 \). If no \( H_1 \) given M0

A1 correct contextual statement follow through from their prob and \( H_1 \). Need faulty bolts and decreased.

NB A correct contextual statement alone followed through from their prob and \( H_1 \) get M1 A1

---

15. (a) The set of values of the test statistic for which the null hypothesis is rejected in a hypothesis test.

Note

1\textsuperscript{st} B1 for “values/ numbers”

2\textsuperscript{nd} B1 for “reject the null hypothesis” o.e or the test is significant

(b) \( X \sim B(30,0.3) \)

\[ P(X \leq 3) = 0.0093 \]

\[ P(X \leq 2) = 0.0021 \]

\[ P(X \geq 16) = 1 - 0.9936 = 0.0064 \]

\[ P(X \geq 17) = 1 - 0.9979 = 0.0021 \]

Critical region is \((0 \leq x \leq 2 \text{ or } 16 \leq x \leq 30)\)
Note
M1 for using B(30,0.3)

1st A1 P(X ≤ 2) = 0.0021

2nd A1 0.0064

3rd A1 for (X) ≤ 2 or (X) < 3 They get A0 if they write
P(X ≤ 2/ X ≤ 3)

4th A1 (X) ≥ 16 or (X) > 15 They get A0 if they write
P(X ≥ 16 X ≥ 15)

NB these are B1 B1 but mark as A1 A1

16 ≤ X ≤ 2 etc is accepted
To describe the critical regions they can use any letter
or no letter at all. It does not have to be X.

(c) Actual significance level 0.0021+0.0064=0.0085 or 0.85%

Note
B1 correct answer only

(d) 15 (it) is not in the critical region
not significant
No significant evidence of a change in P = 0.3
accept H₀, (reject H₁)
P(x ≥ 15) = 0.0169

Note
Follow through 15 and their critical region
B1 for any one of the 5 correct statements up
to a maximum of B2
– B1 for any incorrect statements
16. $H_0: \lambda = 2.5$ (or $\lambda = 5$)  \hspace{1cm} H_1: \lambda < 2.5$ (or $\lambda < 5$) \hspace{1cm} \lambda$ or $\mu$ \hspace{1cm} B1B1

$X \sim \text{Po}(5)$ \hspace{1cm} M1

$P(X \leq 1) = 0.0404$ or $\text{CR } X \leq 1$ \hspace{1cm} A1

$[0.0404 < 0.05]$ this is significant or reject $H_0$ or it is in the critical region \hspace{1cm} M1

There is evidence of a decrease in the (mean) number/rate of deformed blood cells \hspace{1cm} A1

Note

1st B1 for $H_0$ must use lambda or mu; 5 or 2.5.

2nd B1 for $H_1$ must use lambda or mu; 5 or 2.5

1st M1 for use of Po(5) may be implied by probability (must be used not just seen)

\hspace{1cm} eg. $P(X = 1) = 0.0404$ – … would score M1 A0

1st A1 for 0.0404 seen or correct CR

2nd M1 for a correct statement (this may be contextual) comparing their probability and 0.05 (or comparing 1 with their critical region). Do not allow conflicting statements.

2nd A1 is not a follow through. Need the word decrease, number or rate and deformed blood cells for contextual mark.

If they have used $\neq$ in $H_1$ they could get B1 B0 M1 A1 M1A0 mark as above except they gain the

1st A1 for $P(X \leq 1) = 0.0404$ or $\text{CR } X \leq 0$

2nd M1 for a correct statement (this may be contextual) comparing their probability and 0.025 (or comparing 1 with their critical region)

They may compare with 0.95 (one tail method) or 0.975 (one tail method) Probability is 0.9596.

[6]
17. (a) \(X \sim B(20, 0.3)\)

\[
P(X \leq 2) = 0.0355 \quad A1
\]

\[
P(X \leq 9) = 0.9520 \quad \text{so} \quad P(X \geq 10) = 0.0480 \quad A1
\]

Therefore the critical region is \(\{X \leq 2\} \cup \{X \geq 10\}\) \(A1A1\) \(5\)

**Note**

M1 for \(B(20,0.3)\) seen or used

1\(^{\text{st}}\) A1 for 0.0355

2\(^{\text{nd}}\) A1 for 0.048

3\(^{\text{rd}}\) A1 for \(X \leq 2\) or \((X) < 3\) or \([0,2]\) They get A0 if they write \(P(X \leq 2)/X < 3)\)

4\(^{\text{th}}\) A1 \((X) \geq 10\) or \((X) > 9\) or \([10,20]\) They get A0 if they write \(P(X \geq 10/X > 9)\) \(10 \leq X \leq 2\) etc is accepted

To describe the critical regions they can use any letter or no letter at all. It does not have to be \(X\).

(b) \(0.0355 + 0.0480 = 0.0835\) awrt (0.083 or 0.084) \(B1\) \(1\)

**Note**

B1 correct answer only

(c) 11 is in the critical region \(B1ft\)

there is evidence of a change/increase in the proportion/number of customers buying single tins \(B1ft\) \(2\)

**Note**

1\(^{\text{st}}\)B1 for a correct statement about 11 and their critical region.

2\(^{\text{nd}}\) B1 for a correct comment in context consistent with their CR and the value 11

**Alternative**

1\(^{\text{st}}\) B0 \(P(X \geq 11) = 1 - 0.9829 = 0.0171\) since no comment about the critical region

2\(^{\text{nd}}\) B1 a correct contextual statement. \([8]\)
18. (a) \(X \sim B(20, 0.3)\)  
\[P(X \leq 2) = 0.0355\]  
\[P(X \geq 11) = 1 - 0.9829 = 0.0171\]  
Critical region is \((X \leq 2) \cup (X \geq 11)\)  

(b) Significance level = 0.0355 + 0.0171, = 0.0526 or 5.26%  

(c) Insufficient evidence to reject \(H_0\) Or sufficient evidence to accept \(H_0\) /not significant  
\[x = 3\] (or the value) is not in the critical region or 0.1071> 0.025  
Do not allow inconsistent comments

19. (a) (i) \(H_0 : \lambda = 7\) \(H_1 : \lambda > 7\)  
\[X = \text{number of visits.} \sim \text{Po}(7)\]  
\[P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.0985 = 0.1695\]  
\[P(X \leq 9) = 0.1695\]  
CR \(X \geq 11\)  
0.1695 > 0.10, CR \(X \geq 11\)  
Not significant or it is not in the critical region or  
do not reject \(H_0\)  
The rate of visits on a Saturday is not greater/ is unchanged

(ii) \(X = 11\)  

(b) (The visits occur) randomly/ independently or singly or constant rate  

(c) \([H_0 : \lambda = 7\) \(H_1 : \lambda > 7\) (or \(H_0 : \lambda = 14\) \(H_1 : \lambda > 14\)]  
\[X \sim \text{N};(14,14)\]  
\[P(X \geq 20) = P \left( z \geq \frac{19.5 - 14}{\sqrt{14}} \right) \]  
+/- 0.5, stand  
\[= P(z \geq 1.47)\]  
\[= 0.0708\]  
or \(z = 1.2816\)

0.0708 < 0.10 therefore significant. The rate of visits is greater on a Saturday
20. (a) \( X \sim \text{Po}(9) \) may be implied by calculations in part a or b \( M1 \)

\[
\begin{align*}
P(X \leq 3) &= 0.0212 \\
P(X \geq 16) &= 0.0220
\end{align*}
\]

CR \( X \leq 3; \cup X \geq 16 \) \( A1;A1 \) 3

M1 for using Po (9) – other values you might see which imply Po (9) are 0.0550, 0.0415, 0.9780, 0.9585, 0.9889, 0.0111, 0.0062 or may be assumed by at least one correct region.

A1 for \( X \leq 3 \) or \( X < 4 \) condone c1 or CR instead of \( X \)

A1 for \( X \geq 16 \) or \( X > 15 \)

They must identify the critical regions at the end and not just have them as part of their working. Do not accept \( P(X \leq 3) \) etc gets A0

(b) \[
P(\text{rejecting } H_0) = P(X \leq 3) + P(X \geq 16) = 0.0212 + 0.0220 = 0.0432 \text{ or } 0.0433 \]

if they use 0.0212 and 0.0220 they can gain these marks regardless of the critical regions in part a. If they have not got the correct numbers they must be adding the values for their critical regions. (both smaller than 0.05)

You may need to look these up. The most common table values for \( \lambda = 9 \) are in this table

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0062</td>
<td>0.0212</td>
<td>0.0550</td>
<td>0.1157</td>
<td>0.9585</td>
<td>0.9780</td>
<td>0.9889</td>
<td>0.9947</td>
<td>0.9976</td>
<td></td>
</tr>
</tbody>
</table>

A1 awrt 0.0432 or 0.0433

Special case
If you see 0.0432 / 0.0433 and then they go and do something else with it eg 1 – 0.0432 award M1 A0
21. \[ H_0 : p = 0.3; H_1 : p > 0.3 \]

Let \( X \) represent the number of tomatoes greater than 4 cm: \( X \sim B(40, 0.3) \)

\[
\begin{align*}
P(X \geq 18) &= 1 - P(X \leq 17) \\
P(X \geq 18) &= 1 - P(X \leq 17) = 0.0320 \\
&= 0.0320 \\
\text{CR } X \geq 18 \\
0.0320 < 0.05 \\
\end{align*}
\]

no evidence to Reject \( H_0 \) or it is significant

New fertiliser has increased the probability of a tomato being greater than 4 cm

Or

Dhriti’s claim is true

B1 for correct \( H_0 \), must use \( p \) or \( \pi \)

B1 for correct \( H_1 \) must use \( p \) and be one tail.

B1 using \( B(40, 0.3) \). This may be implied by their calculation

M1 attempt to find \( 1 - P(X \leq 17) \) or get a correct probability.

For CR method must attempt to find \( P(X \geq 18) \) or give the correct critical region

A1 awrt 0.032 or correct CR.

M1 correct statement based on their probability, \( H_1 \) and 0.05 or a correct contextualised statement that implies that.

B1 this is not a follow through conclusion in context. Must use the words increased, tomato and some reference to size or diameter.

This is dependent on them getting the previous M1

If they do a two tail test they may get B1 B0 B1 M1 A1 M1 B0

For the second M1 they must have accept \( H_0 \) or it is not significant or a correct contextualised statement that implies that.

22. (a) (i) A hypothesis test is a mathematical procedure to examine a value of a population parameter proposed by the null hypothesis compared with an alternative hypothesis.

B1 Method for deciding between 2 hypothesis.

(ii) The critical region is the range of values or a test statistic or region where the test is significant that would lead to the rejection of \( H_0 \)

B1 range of values. This may be implied by other words. Not region on its own

B1 which lead you to reject \( H_0 \)

Give the first B1 if only one mark awarded.
(b) Let $X$ represent the number of incoming calls: $X \sim \text{Po}(9)$  

From table:

$P(X \geq 16) = 0.0220$  

$P(x < 3) = 0.0212$  

Critical region ($x \leq 3$ or $x \geq 16$)  

B1 using $\text{Po}(9)$  

M1 attempting to find $P(X \geq 16)$ or $P(X \leq 3)$  

A1 $0.0220$ or $P(X \geq 16)$  

A1 $0.0212$ or $P(X \leq 3)$  

These 3 marks may be gained by seeing the numbers in part c  

B1 correct critical region  

A completely correct critical region will get all 5 marks.  

Half of the correct critical region eg $x \leq 3$ or $x \geq 17$ say would get B1 M1 A0 A1 B0 if the M1 A1 A1 not already awarded.

(c) Significance level $= 0.0220 + 0.0212$  

$= 0.0432$ or $4.32\%$  

B1 cao awrt $0.0432$  

(d) $H_0 : \lambda = 0.45; H_1 : \lambda < 0.45$ (accept : $H_0 : \lambda = 4.5; H_1 : \lambda < 4.5$)  

Using $X \sim \text{Po}(4.5)$  

$P(X \leq 1) = 0.0611$  

CR $X < 0$ awrt $0.0611$  

$0.0611 > 0.05$.  

1 $\geq 0$ or 1 not in the critical region  

M1  

There is evidence to Accept $H_0$ or it is not significant  

B1 cao  

There is no evidence that there are less calls during school holidays.

B1 may use $\lambda$ or $\mu$. Needs both $H_0$ and $H_1$  

M1 using $\text{Po}(4.5)$  

A1 correct probability or CR only  

M1 correct statement based on their probability, $H_1$ and 0.05  

or a correct contextualised statement that implies that.  

B1 this is not a follow through. Conclusion in context.  

Must see the word calls in conclusion  

If they get the correct CR with no evidence of using $\text{Po}(4.5)$ they will get M0 A0  

SC If they get the critical region $X \leq 1$ they score M1 for rejecting $H_0$ and B1 for concluding the rate of calls in the holiday is lower.
23. **One tail test**

**Method 1**

- $H_0: \lambda = 5 (\mu = 2.5)$ may use $\lambda$ or $B_1$
- $\mu$ $B_1$
- $H_1: \lambda > 5 (\mu > 2.5)$ $M_1$

$X \sim Po(2.5)$ may be implied $M_1$

<table>
<thead>
<tr>
<th>$P(X \geq 7) = 1 - P(X \leq 6)$</th>
<th>$[P(X \geq 5) = 1 - 0.8912 = 0.1088]$</th>
<th>$P(X \geq 7)$</th>
<th>$P(X \geq 6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= 1 - 0.9858$</td>
<td>$P(X \geq 6) = 1 - 0.9580 = 0.0420$</td>
<td>C.R. $X \geq 6$</td>
<td>C.R. $X \geq 6$</td>
</tr>
<tr>
<td>$= 0.0142$</td>
<td>wrt 0.0142</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$0.0142 < 0.05$ implies $7 \geq 6$ or $7$ is in critical region or $7$ is significant $M_1$

(Reject $H_0$.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. **or**

The scientists claim is justified

**Method 2**

- $H_0: \lambda = 5 (\mu = 2.5)$ may use $\lambda$ or $B_1$
- $\mu$ $B_1$
- $H_1: \lambda > 5 (\mu > 2.5)$ $B_1$

$X \sim Po(2.5)$ may be implied $M_1$

<table>
<thead>
<tr>
<th>$P(X &lt; 7)$</th>
<th>$[P(X &lt; 5) = 0.8912]$</th>
<th>$P(X &lt; 7)$</th>
<th>$P(X &lt; 6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= 0.9858$</td>
<td>C.R. $X \geq 6$</td>
<td>$P(X &lt; 6) = 0.9580$</td>
<td></td>
</tr>
<tr>
<td>$0.9858 &gt; 0.95$</td>
<td>wrt 0.986</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Reject $H_0$.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. **or**

The scientists claim is justified
Two tail test

Method 1

H₀: λ = 5 (λ = 2.5)  
H₁: λ ≠ 5 (λ ≠ 2.5)

X ~ Po (2.5)

\[P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.9858 = 0.0142\]

\[P(X \geq 7) = 1 - 0.9858 = 0.0142\]

0.0142 < 0.025

(Reject H₀.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria.

or

The scientists claim is justified

Method 2

H₀: λ = 5 (λ = 2.5)  
H₁: λ ≠ 5 (λ ≠ 2.5)

X ~ Po (2.5)

\[P(X < 7) = 0.9858\]

\[P(X < 7) = 0.9858\]

0.9858 > 0.975

(Reject H₀.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria.

or

The scientists claim is justified
24. One tail test

Method 1

\[ \text{H}_0: p = 0.2 \quad \text{B1} \]
\[ \text{H}_1: p > 0.2 \quad \text{B1} \]
\[ X \sim B(5, 0.2) \quad \text{may be implied} \quad \text{M1} \]

\[
P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.9421 = 0.0579 \quad \text{att} \quad P(X \geq 3) = 1 - P(X < 3) = 0.9421 \quad \text{M1}
\]

\[
P(X \geq 4) = 1 - 0.9933 = 0.0067 \quad \text{M1}
\]

\[0.0579 > 0.05 \quad 3 \leq 4 \text{ or 3 is not in critical region or 3 is not significant} \quad \text{A1}\]

(Do not reject \( \text{H}_0 \).) There is insufficient evidence at the 5% significance level that there is an increase in the number of times the taxi/driver is late, or
Linda’s claim is not justified

Method 2

\[ \text{H}_0: p = 0.2 \quad \text{B1} \]
\[ \text{H}_1: p > 0.2 \quad \text{B1} \]
\[ X \sim B(5, 0.2) \quad \text{may be implied} \quad \text{M1} \]

\[
P(X < 3) = 0.9421 \quad \text{att} \quad P(X < 3) = 0.9421 \quad \text{M1A1}
\]

\[
P(X < 4) = 0.9933 \quad \text{M1}
\]

\[0.9421 < 0.95 \quad 3 \leq 4 \text{ or 3 is not in critical region or 3 is not significant} \quad \text{M1}\]

(Do not reject \( \text{H}_0 \).) There is insufficient evidence at the 5% significance level that there is an increase in the number of times the taxi/driver is late, or
Linda’s claim is not justified
Two tail test

Method 1

\( H_0: p = 0.2 \) \hspace{1cm} B1
\( H_1: p \neq 0.2 \) \hspace{1cm} B0

\( X \sim X \sim B(5, 0.2) \) \hspace{1cm} may be implied \hspace{1cm} M1

\[
P(X \geq 3) = 1 - P(X \leq 2)
= 1 - 0.9421
= 0.0579
0.0579 > 0.025
\]

\[
\text{CR} X \geq 4 \hspace{1cm} \text{awrt} \ 0.0579
\]

\[
P(X \geq 3) = 1 - 0.9421 = 0.0579
\]

\[
P(X \geq 4) = 1 - 0.9933 = 0.0067
\]

\[
P(X \geq 4) = 1 - 0.9933 = 0.0067
\]

\( (\text{Do not reject } H_0. ) \) There is insufficient evidence at the 5% significance level that there is an increase in the number of times the taxi/driver is late. \hspace{1cm} B1 \hspace{1cm} 7

or

Linda’s claim is not justified

Method 2

\( H_0: p = 0.2 \) \hspace{1cm} B1
\( H_1: p \neq 0.2 \) \hspace{1cm} B0

\( X \sim X \sim B(5, 0.2) \) \hspace{1cm} may be implied \hspace{1cm} M1

\[
P(X < 3) = 1 - P(X \geq 4)
= 1 - 0.9421
= 0.0975
0.0975 < 0.975
\]

\[
P(X < 3) = 0.9421
\]

\[
P(X < 4) = 0.9933
\]

\( (\text{Do not reject } H_0. ) \) There is insufficient evidence at the 5% significance level that there is an increase in the number of times the taxi/driver is late. \hspace{1cm} B1

or

Linda’s claim is not justified

Special Case

If they use a probability of \( \frac{1}{7} \) throughout the question they may gain \hspace{1cm} B1B1M0M1A0M1B1.

\( \text{NB they must attempt to work out the probabilities using } \frac{1}{7} \)
25. (a) \( H_0 : p = 0.20, H_1 : p < 0.20 \)  
Let \( X \) represent the number of people buying family size bar.  
\( X \sim B (30, 0.20) \)  
\( P(X \leq 2) = 0.0442 \) or \( P(X \leq 2) = 0.0442 \) awrt 0.044  
\( P(X \leq 3) = 0.1227 \)  
CR \( X \leq 2 \)  
0.0442 < 5\%, so significant. Significant  
There is evidence that the no. of family size bars sold is lower than usual.  
A1  6

(b) \( H_0 : p = 0.02, H_1 : p \neq 0.02 \)  
\( \lambda = 4 \) etc ok both  
Let \( Y \) represent the number of gigantic bars sold.  
\( Y \sim B (200, 0.02) \Rightarrow Y \sim Po (4) \)  
\( P(Y = 0) = 0.0183 \) and \( P(Y \leq 8) = 0.9786 \Rightarrow P(Y \geq 9) = 0.0214 \)  
first, either  
Critical region \( Y = 0 \cup Y \geq 9 \)  
\( Y \leq 0 \) ok  
B1,B1  6

N.B. Accept exact Bin: 0.0176 and 0.0202

(c) Significance level = 0.0183 + 0.0214 = 0.0397  
awrt 0.04  
B1  1

[13]

26. (a) Let \( X \) represent the number of breakdowns in a week.  
\( X \sim P_\theta (1.25) \)  
\( \theta = 1.25 \)  
Implied  
\( P(X < 3) = P(0) + P(1) + P(2) \) or \( P(X \leq 2) \)  
\( = e^{-1.25} \left( 1 + 1.25 + \frac{(1.25)^2}{2!} \right) \)  
\( = 0.868467 \)  
awrt 0.868 or 0.8685
(b) H₀: \( \lambda = 1.25 \); H₁: \( \lambda \neq 1.25 \) (or H₀: \( \lambda = 5 \); H₁: \( \lambda \neq 5 \))

Let Y represent the number of breakdowns in 4 weeks

Under H₀, \( Y \sim P(5) \)

\[ P(Y \geq 11) = 1 - P(Y \leq 10) \quad \text{or} \quad P(X \geq 11) = 0.0137 \]

One needed for M

\[ P(X \geq 10) = 0.0318 \]

\[ = 0.0137 \]

CR \( X \geq 11 \)

0.0137 < 0.025, 0.0274 < 0.05, 0.9827 > 0.975, 0.9726 > 0.95 or 11 ≥ 11

Evidence that the rate of breakdowns has changed / decreased

Context

From their p

27. (a) Let X represent the number of bowls with minor defects.

\[ \therefore X \sim B(25, 0.20) \]

\[ P(X \leq 1) = 0.0274 \quad \text{or} \quad P(X = 0) = 0.0038 \]

need to see at least one.

prob for \( X \leq 0 \) For M₁

\[ P(X \leq 9) = 0.9827; \Rightarrow P(X \geq 10) = 0.0173 \]

either

\[ \therefore CR \text{ is } \{X \leq 1 \cup X \geq 10\} \]
(b) Significance level = 0.0274 + 0.0173
\[= 0.0447 \text{ or } 4.477\%\]
awrt 0.0447

\[H_0: p = 0.20; H_1: p < 0.20;\]

Let \(Y\) represent number of bowls with minor defects

Under \(H_0\) \(Y \sim B(20, 0.20)\)

\[= 0.2061\]

\[P(Y \leq 2) \text{ or } P(Y \leq 2) = 0.2061\]

\[P(Y \leq 2) \text{ or } P(Y \leq 2) = 0.2061\]

\[= 0.2061\]

\[CR Y \leq 1 = 0.0692\]

\[0.2061 > 0.10 \text{ or } 0.7939 < 0.9 \text{ or } 2 > 1\]

Insufficient evidence to suggest that the proportion of defective bowls has decreased.

[14]

28. (a) (i) Two tail

\[H_0: p = 0.2, H_1: p \neq 0.2\]

\[p = P(X \geq 9) = 1 - P(X \leq 8) \text{ or attempt critical value/region}\]

\[= 1 - 0.9900 = 0.01\]

\[CR X \geq 9\]

0.01 < 0.025 or \(9 \geq 9\) or \(0.99 > 0.975\) or \(0.02 < 0.05\) or lies in interval with correct interval stated.

Evidence that the percentage of pupils that read Deano is not 20%.

(ii) \(X \sim Bin(20, 0.2)\) may be implied or seen in (i) or (ii)

\[So 0 \text{ or } [9, 20] \text{ make test significant.}\]

0.9, between “their 9” and 20
(b) $H_0 : p = 0.2, H_1 : p \neq 0.2$  
$W \sim \text{Bin}(100, 0.2)$  
$W \sim \text{N}(20, 16)$  

$P(X \leq 18) = P\left(Z \leq \frac{18.5 - 20}{4}\right) \text{ or } P\left(Z \geq \frac{18.5 - 20}{4}\right) = \pm 1.96$  

$\pm \text{cc, standardise or use } z \text{ value, standardise}$ 

$= P(Z \leq -0.375)$ 

$= 0.352 - 0.354$  

$\text{CR } X < 12.16 \text{ or } 11.66 \text{ for } \frac{1}{2}$  

$[0.352 > 0.025 \text{ or } 18 > 12.16 \text{ therefore insufficient evidence to reject } H_0$ 

Combined numbers of Deano readers suggests 20% of pupils read Deano  

(c) Conclusion that they are different.  

Either large sample size gives better result  

Or  

Looks as though they are not all drawn from the same population.  

(a) (i) One tail  

$H_0: p = 0.2, H_1: p \neq 0.2$  

$P(X \geq 9) = 1 - P(X \leq 8) \text{ or attempt critical value/region}$ 

$= 1 - 0.9900 = 0.01$  

$0.01 < 0.025 \text{ or } 9 \geq 9 \text{ or } 0.99 > 0.975 \text{ or } 0.02 < 0.05 \text{ or lies in interval with correct interval stated.}$ 

Evidence that the percentage of pupils that read Deano is not 20%  

(ii) $X \sim \text{Bin}(20, 0.2)$ may be implied or seen in (i) or (ii)  

So 0 or [9,20] make test significant.  

0.9, between “their 9” and 20
(b) \[ H_0 : p = 0.2, \quad H_1 : p \neq 0.2 \]

\[ W \sim \text{Bin}(100, 0.2) \]

\[ W \sim \text{N}(20, 16) \quad \text{normal; 20 and 16} \]

\[ P(X \leq 18) = P(Z \leq \frac{18.5 - 20}{4} \quad \text{or} \quad \frac{x - 20}{4} = -1.6449) \]

\[ \pm \text{cc, standardise or standardise, use } z \text{ value} \]

\[ = P(Z \leq -0.375) \]

\[ = 0.3520 \quad \text{CR } X < 13.4 \text{ or } 12.9 \quad \text{awrt } 0.352 \]

\[ 0.352 > 0.025 \text{ or } 18 > 12.16 \text{ therefore insufficient evidence to reject } H_0 \]

Combined numbers of Deano readers suggests 20% of pupils read Deano

(c) Conclusion that they are different.

Either large sample size gives better result

Or

Looks as though they are not all drawn from the same population.

29. (a) Probability of success/failure is constant

Trials are independent

(b) Let \( p \) represent proportion of students who can distinguish between brands

\[ H_0 : p = 0.1; \quad H_1 : p > 0.1 \]

\[ \text{both} \]

\[ \alpha = 0.01; \quad \text{CR: } \delta > 2.3263 \]

\[ 2.3263 \]

\[ np = 25; \quad npq = 22.5 \]

\[ \text{both} \]

\[ \text{Can be implied} \]

\[ \delta = \frac{39.5 - 25}{\sqrt{22.5}} = 3.0568…. \]

\[ \text{Standardisation with } \pm 0.5 \text{ their } \sqrt{npq} \]

\[ \text{AWRT } 3.06 \]

Reject \( H_0: \) claim cannot be accepted

Based on clear evidence from \( \delta \) or \( p \)
(c) \[\text{eg: } np, nq \text{ both 75 – true or acceptable}\]
\[p \text{ close tp } 0.5 \text{ – not true, assumption not met}\]
\[\text{success/failure not clear cut necessarily}\]
\[\text{independence – one student influences another}\]

\[\text{[10]}\]

(b) \[\text{\underline{Aliter}} \quad \delta = 3.06 \Rightarrow p = 0.9989 > 0.99\]
\[\text{or } p = 0.0011 < 0.01\]
\[\text{B1 eqn to 2.3263}\]

30. Let \(X\) represent number of accidents/month \(\sim P(3)\)

(a) \[P(X > 4) = 1 - P(X \leq 4); = 1 - 0.8513 = 0.1847\]
\[\text{M1; A1} \quad 3\]

(b) Let \(Y\) represent number of accidents in 3 months
\[\sim P(3 \times 3 = 9)\]
\[\text{B1}\]
\[\text{Can be implied}\]
\[P(Y > 4) = 1 - 0.0550 = 0.9450\]
\[\text{B1} \quad 2\]

(c) \(H_0: \lambda = 3; H_1: \lambda < 3\)
\[\alpha = 0.05\]
\[P(X \leq 1/\lambda = 3) = 0.1991; > 0.05\]
\[\text{B1 M1}\]
\[\text{detailed; allow } B0B1M1 (0.025) A0\]
\[\therefore \text{Insufficient evidence to support the claim that the mean}\]
\[\text{number of accidents has been reduced.}\]
\[\text{A1ft} \quad 4\]

(NB: CR: \(X \leq 0; X = 1\) not in CR; same conclusion \(\Rightarrow B1, M1, A1\))
(d) \( H_0: \lambda = 24 \times 3 = 72; H_1: \lambda < 72 \)  
\[ \text{can be implied } \lambda = 72 \]
\[ \alpha = 0.05 \Rightarrow CR: \delta < -1.6449 \]
\[ \text{both } H_0 \text{ & } H_1 \]
\[ -1.6449 \]

Using Normal approximation with \( \mu = \sigma^2 = 72 \)
\[ \text{Can be implied} \]
\[ \delta = \frac{55.5 - 72}{\sqrt{72}} = -1.94454… \]
\[ \text{Stand. with } \pm 0.5, \mu = \sigma \]
\[ \text{AWRT } -1.94/5 \]

Since \(-1.944…\) is in the CR, \(H_0\) is rejected.  
There is evidence that the restriction has reduced the number of accidents

Context & clear evidence

Aliter (d)
\[ p = 0.0262 < 0.05 \]
\[ \text{AWRT } 0.026 \text{ equn to } -1.6449 \]

31. \( H_0: p = 0.25, H_1 = p > 0.25 \)  
\[ \text{1 tailed} \]
Under \( H_0, X \sim \text{Bin}(25, 0.25) \)
\[ \text{Implied by probability} \]
\[ P(X \geq 10) = 1 - P(X \leq 9) = 0.0713 > 0.05 \]
\[ \text{Correct inequality, } 0.0713 \]
Do not reject \( H_0 \), there is insufficient evidence to support Brad’s claim.

DNR, context

32. (a) A range of values of a test statistic such that if a value of the test statistic obtained from a particular sample lies in the critical region, then the null hypothesis is rejected (or equivalent).
(b) \[ P(X < 2) = P(X = 0) + P(X = 1) \] 
\[ = e^{\frac{1}{7}} + e^{\frac{2}{7}} \]
both M1
\[ = 0.990717599... = 0.9907 \text{ to } 4 \text{ sf} \]
awrt 0.991
\[ X \sim P(14 \times \frac{1}{7}) = P(2) \]
both A1
\[ \text{Correct inequality, } 0.9473 \]
\[ \text{H}_0: \lambda = 4, \text{ H}_1: \lambda < 4 \]
B1
\[ \text{Accept } \mu \text{ & } H_0: \lambda = \frac{1}{7}, H_1: \lambda < \frac{1}{7} \]
B1
\[ X \sim P(4) \]
Implied
\[ P(X \leq 1) = 0.0916 > 0.05, \]
Inequality 0.0916
So insufficient evidence to reject null hypothesis
A1
Number of breakdowns has not significantly decreased
A1 7

33. (a) Vehicles pass at random / one at a time / independently /
at a constant rate Any 2\&context
B1B1dep 2
(b) \[ X \text{ is the number of vehicles passing in a 10 minute interval,} \]
\[ X \sim \text{Po} \left( \frac{51}{60} \times 10 \right) = \text{Po}(8.5) \]
B1
Implied Po(8.5)
\[ P(X = 6) = \frac{8.5^6 e^{-8.5}}{6!}, = 0.1066 \text{ (or } 0.2562 - 0.1496 = 0.1066) \]
M1A1 3
Clear attempt using 6, 4dp
(c) \[ P(X \geq 9) = 1 - P(X \leq 8) = 0.4769 \]
M1A1 2
Require 1 minus and correct inequality
34. (a) Let $X$ represent the number of plant pots with defects, $X \sim B(20, 0.20)$

Implied

$P(X \leq 1) = 0.0274$, $P(X \geq 10) = 0.0173$

Clear attempt at both tails required, 4dp

Critical region is $X \leq 1, X \geq 10$

(b) Significance level $= 0.0274 + 0.0173 = 0.0447$

Accept % 4dp

(c) $H_0: \lambda = 10, H_1: \lambda > 10$ (or $H_0: \lambda = 60, H_1: \lambda > 60$)

Let $Y$ represent the number sold in 6 weeks, under $H_0$, $Y \sim Po(60)$

$P(Y \geq 74) \approx P(W > 73.5)$ where $W \sim N(60, 60)$

$\approx P(Z \geq \frac{73.5 - 60}{\sqrt{60}}) = P(Z > 1.74) = 0.047 - 0.0409 < 0.05$

Standardise using $60\sqrt{60}$

Evidence that rate of sales per week has increased.

35. (a) $X \sim Po(7)$

$P(X \leq 2) = 0.0296$

$P(X \geq 13) = 1 - 0.9370 = 0.0270$

Critical region is $(X \leq 2) \cup (X \geq 13)$

(b) Significance level $= 0.0296 + 0.0270 = 0.0566$

(c) $x = 5$ is not the critical region $\Rightarrow$ insufficient evidence to reject $H_0$
36. (a) \( X = \) no. of vases with defects \( \sim B(20, 0.15) \)
\[
P(X \leq 0) = 0.0388
\]

**Use of tables to find each tail**

\[
P(X \leq 6) = 0.9781 \quad \therefore P(X \geq 7) = 0.0219
\]
\[
\therefore \text{critical region is } X \leq 0, \quad \text{or } X \geq 7
\]

Significance level = \( P(X \leq 0) + P(X \geq 7) = 0.0388 + 0.0219 = 0.0607 \) (B1) 1

\( H_0: \lambda = 2.5, \quad H_1: \lambda > 2.5 \) \[ \text{or } H_0: \lambda = 10, \quad H_1: \lambda > 10 \]

\( Y = \) no. sold in 4 weeks. \quad Under \( H_0 \) \( Y \sim Po(10) \)
\[
P(Y \geq 15) = 1 - P(Y \leq 14) = 1 - 0.9165 = 0.0835
\]

More than 5% so not significant. Insufficient evidence of an increase in the rate of sales.

1. This was poorly done with very few candidates scoring full marks. Those candidates who had learnt standard definitions fared better than those who used their own understanding of the terms because they were less likely to leave out vital elements of the definitions. Even those who answered parts (a) and (b) correctly were then unable to apply these definitions in context.

In part (a) a large majority of candidates omitted to mention “all”, or its equivalent.

Part (b) was well answered because many candidates used a standard definition. The most common errors were using “population” instead of “sample and omitting “no unknown parameters”.

In part (c) a substantial number of candidates were confused about “the population in this case”. Many thought it to be the sample of 100 voters. Others were closer to the truth with “all the residents of the town”, but did not earn the mark because they had failed to distinguish between registered voters and residents. The statistic was more easily identified.

Part (d) was poorly answered with many candidates having no idea what a sampling distribution was and those that did being unable to put it into context. The sampling distribution of a proportion is arguably one of the hardest to get a grip on and articulate convincingly.

2. A high proportion of candidates attempted the first two parts of this question successfully, with the majority of candidates getting at least one mark for part (b). Those less successful in part (a) either misread the question and ended up with a denominator of 3 for the probabilities or confused formulae for calculating the mean and variance and used, for example, \( \sum \frac{xP(x)}{n} \) for the mean or used \( E(X^2) \) for \( \sigma^2 \). The solution to part (c) proved beyond the capability of a minority of candidates but, for the majority, many exemplary answers were evident, reflecting sound preparation on this topic. Candidates who found all 8 cases in (b) usually gained four marks in part (c) for calculating the probabilities. For a small percentage of those candidates, calculating the means was difficult and hence completing the table correctly was not possible. A few candidates tried unsuccessfully to use the binomial to answer part (c).

3. This question was either answered very well with some text book solutions, although it seemed that only a minority of candidates earned all five marks, or badly with some strange descriptions. A reasonable number of candidates responded with comments that were very close to those in the mark scheme: evidence possibly of deliberate preparation and learning whilst others had internalised the concepts and provided responses in their own words.

Whilst these responses might not have matched the ‘official’ answers, they nevertheless captured the essence of the concepts and were fully acceptable. There was confusion with the definition of statistics and parameters and part (b) was often attempted badly with candidates not knowing the definition of a probability distribution. On the whole this was one of the worst answered questions in the paper.

In part (a) candidates gave various definitions sometimes all muddled up. Not many candidates gave clear definitions but a common error was candidates writing “any function” or “no other quantities”.

In part (b) again the candidates had mixed success. A significant minority scored marks by knowing that a sampling distribution involved all possible values of the statistic and their associated probabilities.

In part (c) many could identify (ii) correctly and a variety of reasons were seen. This part seemed to be done well
even by candidates who could not answer part (a). It was interesting to see that a relatively large proportion of candidates who earned both marks for part (c), were unable to achieve either of the two marks in part (a). There was a connection between parts (a) and (c) that many candidates failed to recognise. If those candidates who wrote “(ii) is not a statistic because it has unknown parameters” had then reflected on their responses to parts (a) and (c), they could then have gone back to modify their answer to (a) in order to earn more marks.

4. Nearly all candidates achieved at least one of the available marks but it was disappointing that there were not more attaining full marks.
   (a) Too many candidates referred to the national census rather than a general definition. Some felt an enumeration was adequate and others failed to recognise that EVERY member had to be investigated.
   (b) A failure to put the question in context and consider the consequences of testing every item meant that some candidates scored 0 in this part of the question. A few candidates did not read the question carefully and used cheap and quick as their reasons why a census should not be used when the question specifically said give a reason “other than to save time and cost”.
   (c) Many candidates mentioned a list; database or register and so attained the available mark. However, some did not seem to differentiate between the population and the sampling frame.
   (d) Most candidates were able to identify the sampling units correctly, although those who had not scored in part (c) tended to say: “the sample of 5 cookers” in part (d).

5. Almost all candidates answered part (a) correctly, a minority failed to mention “census” or “asking all members” when answers referred to long time/expensive/difficult. In part (b) many candidates failed to include the word “all” in their answer. Quite a number did not know or understand the term sampling frame and wrote about sampling methods. Most candidates answered part (c) correctly, but there were occasional references to golfers rather than members or to those selected in the sample.

6. In part (a) many candidates were able to calculate the mean accurately, although some divided by random constants. Few drew up a table and many were unable to cope with the 5p coins. The most common error in calculating the variance was the failure to subtract \(E(X)^2\). Most candidates correctly identified 6 possible samples but some failed to realise that combinations such as (1,5) and (5,1) were different and so missed the other 3 possibilities. Only a minority of candidates were able to attempt part (c) of the question with any success, with many candidates having no idea what was meant by ‘the sampling distribution of the mean value of the samples’. Most did not find the mean values and if they did, then they were unable to find the probabilities (ninths were common). Very few candidates achieved full marks.

7. This question proved difficult to many candidates. Errors in this part (a) included the use of the word sample rather than population. Many candidates also gave an ambiguous response to part (b), often omitting to mention all sampling units or the whole population. Part (c) was done badly and whilst some candidates scored 1 mark very few achieved both marks. It appeared that many candidates had attempted to memorise the definition, but it came out garbled and confused with other concepts.

8. The bookwork required to answer this question was not remembered as well as it should have been. Many candidates could not define a population or a sampling frame in detail or know why they might be different. In part (c) many candidates were unable to give in sufficient detail a justified example of the use of a census and a sample.

9. Weaker students had difficulties with this question with a considerable number scoring 1 or 0 marks. In part (a) good candidates answered this correctly but for many there was confusion between a population and a sample and that the population must be in a list or equivalent. In part (b) those candidates who had learnt the basic definitions were able to answer this successfully.

10. Only a very few candidates achieved full marks. Most scored 2 or 3 out of the 5 available. Common errors were in part (c) where only a very small number could provide a valid disadvantage and in part (d) not all candidates realised the problem of having an incomplete (or not up-to-date) sampling frame.

11. This question also allowed candidates to score highly; indeed some otherwise poor papers were redeemed by good marks here. Most marks were lost in the opening parts where it is clear that candidates do not understand well enough the need for a degree of precision in defining terms such as population and sampling frame. Similarly it is a cause for concern that the majority of candidates talk about a census giving more accurate answers (even though this was allowed) rather than understanding the real differences between a sample and a census. Part (e) received a very high
number of correct answers, and part (f), although less well done, did receive an encouragingly high number of good
solutions, with context being well used. The most common mistakes were careless statements of the hypotheses and a
decision to find $P(X = 6)$. Part (g) was very well answered with a large number of candidates gaining full marks.
Very few candidates used incorrect parameters in the normal approximation, but the most common cause of loss of
marks was in an error in the use of either 70.5 or 82.5 even if a correct probability statement had been given earlier.

12. No Report available for this question.

13. The majority of candidates were familiar with the technical terms in part (a), but failed to establish any context.
Part (b) was a useful source of marks for a large proportion of the candidates. The only problems were occasional
errors in detail. In part (i) a few did not spot the change in time scale and used Po(4) rather than Po(8). Some were
confused by the wording and calculated $P(X = 8)$ rather than $P(X = 0)$. The main source of error for (ii) was to find $1
- P(X \leq 4)$ instead of $1 - P(X \leq 3)$.

In part (c) the Normal distribution was a well-rehearsed routine for many candidates with many candidates
concluding the question with a clear statement in context.
The main errors were

- Some other letter (or none) in place of $\lambda$ or $\mu$
- Incorrect Normal distribution: e.g. $\text{N}(60, 60)$
- Omission of (or an incorrect) continuity correction
- Using 48 instead of 60
- Calculation errors

A minority of candidates who used the wrong distribution (usually Poisson) were still able to earn the final two marks
in the many cases when clear working was shown. This question was generally well done with many candidates
scoring full marks.

14. Part (a) was well answered as no context was required.
In part (b) candidates identified the correct distribution and with much of the working being correct. However
although the lower limit for the critical region was identified the upper limit was often incorrect. It is disappointing
to note that many candidates are still losing marks when they clearly understand the topic thoroughly and all their work
is correct except for the notation in the final answer. It cannot be overstressed that $P(X \leq 6)$ is not acceptable
notation for a critical region. Others gave the critical region as $19 \leq X \leq 19$.

In part (c) the majority of candidates knew what to do and just lost the accuracy mark because of errors from part (b)
carried forward.
Part (d) tested the understanding of what a critical region actually is, with candidates correctly noting that 8 was
outside the critical region but then failing to make the correct deduction from it. Some were clearly conditioned to
associate a claim with the alternative hypothesis rather than the null hypothesis. A substantial number of responses
where candidates were confident with the language of double-negatives wrote “8 is not in the critical region so there
is insufficient evidence to disprove the company’s claim”. Other candidates did not write this, but clearly understood
when they said, more simply “the company is correct”.
Part (e) was generally well done with correct deductions being made and the contextual statement being made. A few
worked out $P(X = 5)$ rather than $P(X \leq 5)$.

15. Part (a) tested candidates’ understanding of the critical region of a test statistic and responses were very varied, with
many giving answers in terms of a ‘region’ or ‘area’ and making no reference to the null hypothesis or the test being
significant. Many candidates lost at least one mark in part (b), either through not showing the working to get the
probability for the upper critical value, i.e. $1 - P(X \leq 15) = P(X \geq 16) = 0.0064$, or by not showing any results that
indicated that they had used $\text{B}(30, 0.3)$ and just writing down the critical regions, often incorrectly. A minority of
candidates still write their critical regions in terms of probabilities and lose the final two marks. Responses in part (c)
were generally good with the majority of candidates making a comment about the observed value and their critical
Candidates seemed better prepared for this type of question than in previous years. Marks were often lost for not using $\lambda$ or $\mu$ in the hypotheses and for not putting the conclusion into context. A significant minority of candidates found $P(X = 1)$ instead of $P(X \leq 1)$ but only a few candidates chose the critical region route.

This was a very well answered question. Candidates were able to use binomial tables and gave the answer to the required number of decimal places. As in previous years there were some candidates who confused the critical region with the probability of the test statistic being in that region but this error has decreased. Candidates were able to describe the acceptance of the hypothesis in context although sometimes it would be better if they just repeated the wording from the question which would help them avoid some of the mistakes seen. There were still a few candidates who did not give a reason in context at all.

In part (a) many candidates failed to read this question carefully assuming it was identical to similar ones set previously. Most candidates correctly identified $B(20,0.3)$ to earn the method mark and many had 0.0355 written down to earn the first A mark, although in light of their subsequent work, this may often have been accidental. A majority of candidates did not gain the second A mark as they failed to respond to the instruction “state the probability of rejection in each case”. In the more serious cases, candidates had shown no probabilities from the tables, doing all their work mentally, only writing their general strategy: “$P(X \leq c) < 0.05$”. Whilst many candidates were able to write down the critical region using the correct notation there are still some candidates who are losing marks they should have earned, by writing $P(X \leq 2)$ for the critical region $X \leq 2$

Part (b) was usually correct.

Part (c) provided yet more evidence of candidates who had failed to read the question: “in the light of your critical region”. Some candidates chose not to mention the critical region and a number of those candidates who identified that 11 was in the critical region did not refer to the manager’s question.

Part (a) of this question was poorly done. Candidates would appear unfamiliar with the standard mathematical notation for a Critical Region. Thus $11 \leq X \leq 2$ made its usual appearances, along with $c_1 = 2$ and $P(X \leq 2)$

In part (b) candidates knew what was expected of them although many with incorrect critical regions were happy to give a probability greater than 1 for the critical region.

Part (c) was well answered. A few candidates did contradict themselves by saying it was “significant” and “there is no evidence to reject $H_0$” so losing the first mark.

In Part (a) there are a sizeable number of candidates who are not using the correct symbols in defining their hypotheses although the majority of candidates recognised Po(7).

For candidates who attempted a critical region there were still a number who struggled to define it correctly for a number of reasons:

- Looking at the wrong tail and concluding $X\leq3$.
- Incorrect use of $>$ sign when concluding 11 - not appreciating that this means $\geq12$ for a discrete variable.
- Not knowing how to use probabilities to define the region correctly and concluding 10 or 12 instead of 11.

The candidates who opted to calculate the probability were generally more successful.

A minority still try to calculate a probability to compare with 0.9. This proved to be the most difficult route with the majority of students unable to calculate the probability or critical region correctly. We must once again advise that this is not the recommended way to do this question. There are still a significant number who failed to give an answer in context although fewer than in previous sessions.

Giving the minimum number of visits needed to obtain a significant result proved challenging to some and it was noticeable that many did not use their working from part (a) or see the connection between the answer for (i) and (ii) and there were also number of candidates who did not recognise inconsistencies in their answers.

A number of candidates simply missed answering part (b) but those who did usually scored well.

There were many excellent responses in part (c) with a high proportion of candidates showing competence in using a Normal approximation, finding the mean and variance and realising that a continuity correction was needed. Marks were lost, however, for not including 20, and for not writing the conclusion in context in terms of the rate of visits being greater. Some candidates attempted to find a critical value for $X$ using methods from S3 but failing to use 1.2816. There were a number of candidates who calculated $P(X = 20)$ in error.
20. Whilst many candidates knew what they were doing in part (a) they lost marks because they left their answers as \( P(X \leq 3) \) etc and did not define the critical regions. A few candidates were able to get the figures 0.0212 and 0.0220 but then did not really understand what this meant in terms of the critical value. A critical region of \( X \geq 15 \) was common. Part (b) was poorly answered. The wording “incorrectly rejecting \( H_0 \)” confused many candidates. They often managed to get to 0.432 but then they took this away from 0.5 or occasionally 1. It was not uncommon for this to be followed by a long paragraph trying to describe what they had done.

21. The majority of candidates appeared to have coped with this question in a straightforward manner and made good attempts at a conclusion in context, which was easily understood.

The hypotheses were stated correctly by most candidates – they seem more at ease with writing “\( \mu = \)” than in Q7 where \( \lambda \) is the parameter. Most used the correct distribution B(40, 0.3). Those who stated the correct inequality usually also found the correct probability/critical region and thus rejected \( H_0 \). The main errors were to calculate \( 1 - P(X \leq 18) \) or \( P(X = 18) \). Some candidates used a critical region approach but the majority calculated a probability. A minority of candidates still attempted to find a probability to compare with 0.95. This was only successful in a few cases and it is recommended that this method is not used. Most candidates who took this route found \( P(X \leq 18) \) rather than \( P(X \leq 17) \). There were difficulties for some in expressing an accurate contextualised statement. The candidates who used a critical region method here found it harder to explain their reasoning and made many more mistakes.

22. This question appeared to be difficult for many candidates with a large proportion achieving less than half the available marks.

(a) The majority of candidates were unable to give an accurate description of a hypothesis test as a method of deciding between 2 hypotheses. There were more successful definitions of a critical region but many candidates achieved only 0 or 1 of the 3 available marks. Common errors included too much re-use of the word region without any expansion on it. Even those who could complete the rest of the question with a great deal of success could not describe accurately what they were actually doing.

(b) Although most of those attempting this part of the question realised that a Poisson distribution was appropriate there was a sizeable number who used a Binomial distribution. Again, the most common problem was in expressing and interpreting inequalities in order to identify the critical regions. Many found the correct significance level but struggled to express the critical region correctly. Answers with 15 were common and some candidates even decided that 4 to 15 was the CR.

(c) Those candidates that identified the correct critical regions were almost always able to state the significance level correctly, as were some who had made errors in stating these regions. Some still gave 5% even with part (b) correct.

(d) Candidates who had used a Binomial distribution in part (b), and many of those who had not, used \( \mu \) instead of \( \lambda \) in stating the hypotheses and went on to obtain a Binomial probability in this part of the question. In obtaining \( P(X \leq 1) \) some used Po(9) from part (b) instead of Po(4.5). Most of those achieving the correct statement (failing to reject \( H_0 \)) were able to place this in a suitable contextualised statement. There were some candidates who still tried to find \( P(X = 1) \) rather than \( P(X < 1) \).

23. The majority of candidates found this question straightforward. They were most successful if they used the probability method and compared it with 0.05. Those who attempted to use 95% were less successful and this is not a recommended route for these tests.

Most candidates knew how to specify the hypotheses with most candidates using 2.5 rather than 5. Some candidates used \( \mu \), or did not use a letter at all, in stating their hypotheses, but most of the time they used \( \lambda \). A minority found \( P(X = 7) \) and some worked with Po(5).

If using the critical region method, not all candidates showed clearly, either their working, or a comparison with the value of 7 and the CR \( X \geq 7 \). A sizeable minority of candidates failed to put their conclusion back into the given context. Reject \( H_0 \) is not sufficient.

24. There was clear evidence that candidates had been well prepared for a question on hypothesis testing with many candidates scoring full marks on this question. Candidates who used the probability method were generally more successful than those who used critical regions. They were less familiar with writing hypotheses for \( \mu \) than for the mean and so used \( \lambda \) or \( \mu \) instead of \( \mu \). A few candidates mistakenly used a B(5, 1/7) or B(7,1/7) distribution. In a minority of cases the final mark was lost through not writing the conclusion in context using wording from the question.
25. Weak candidates found this question difficult and even some otherwise very strong candidates failed to attain full marks. Differentiating between hypothesis testing and finding critical regions and the statements required, working with inequalities and placing answers in context all caused problems. In part (a) a large number of candidates were able to state the hypotheses correctly but a sizeable minority made errors such as missing the \( p \) or using an alternative (incorrect) symbol. Some found \( P(X = 2) \) instead of \( P(X \leq 2) \) and not all were able to place their solution in the correct context. Not all candidates stated the hypotheses they were using to calculate the critical regions in part (b). In a practical situation this makes these regions pointless. The lower critical region was identified correctly by many candidates but many either failed to realise that \( P(X \leq 8) = 0.9786 \) would give them the correct critical region and/or that this is \( X \geq 9 \). The final part was often correct.

26. Most candidates answered Part (a) correctly. A small number of candidates calculated the probability for less than or equal to 3 although a minority thought that dividing by 0! in \( P(X = 0) \) gave zero. In part (b) carrying out the hypothesis test was more challenging though there was clear evidence that candidates had been prepared for this type of question. However, using \( p \) instead of \( \lambda \) or \( \mu \), when stating the hypotheses, was often seen and incorrectly stating \( H_1 \) as \( \lambda > 1.25 \) or 5 also lost marks. Many candidates calculated \( P(X \leq 11) \) instead of looking at \( P(X \geq 11) \). A diagram would have helped them or the use of the phrase “a result as or more extreme than that obtained”. Those who used the critical region approach made more errors. Some candidates correctly calculated the probability and compared it with 0.025 but were then unsure of the implications for the hypotheses. A few candidates used a 2-tailed hypothesis but then used 0.05 rather than 0.025 in their comparison. Most candidates gave their conclusions in context.

27. Part (a) was one of the poorest answered questions in the paper. Many candidates quoted the inequalities with little or no understanding of how to apply them and too many merely stated the critical values with no figures to back them up and without going on to give the critical region. It was unclear in some cases whether they knew that the critical region was the two tails rather than the central section. A few candidates used diagrams and this almost always enabled them to give a correct solution. Many misunderstood the wording of the question and thought that one of the tails could be slightly larger than 2.5%. Those that got Part (a) correct usually got part (b) correct, although a minority of weaker candidates did not understand what was meant by significance level. Part (c) was well answered. Those candidates who used the critical region approach did less well, tending to get themselves muddled. A few did not make the correct implication at the end and too many did not state that 0.2061 > 0.10 but merely said the result was not significant. The context for accepting/rejecting the null hypothesis was not always given.

28. In part (a)(i) the null and alternative hypotheses were stated correctly by most candidates but then many had difficulties in either calculating the probability or obtaining the correct critical region and then comparing it to the significance level or given value. Most of those obtaining a result were able to place this in context but not always accurately or fully. Candidates still do not seem to realise that just saying accept or reject the hypothesis is inadequate.
In (a)(ii) although some candidates obtained the critical regions the list of values was not always given. Many candidates got the 9 but forgot the 0 and a minority gave a value of \( \geq 9 \) but did not give the upper limit.

In part (b) there was a wide variety of errors in the solutions provided including using the incorrect approximation, failing to include the original sample in the calculations, not using a continuity correction and errors in using the normal tables. Again in this part many candidates lost the interpretation mark. Most candidates attempting part (c) of the question noted that the results for the two hypothesis tests were different but few suggested that either the populations were possibly not the same for the samples or that larger samples are likely to yield better results.

29. Most candidates wrote down two other conditions associated with the binomial experiment but too many did not use ‘trials’ when referring to independence. The alternative hypothesis was often wrongly defined and far too many of those using the normal approximation ignored the need to use the continuity correction. The conclusion needed to be in context but many did not do this. Few candidates made any sensible attempt to answer part (c).

30. For those candidates that could interpret ‘more than 4 accidents occurred’ correctly parts (a) and (b) were a good source of marks. Part (b) was often well answered and many candidates gained full marks. In part (c) incorrect hypotheses and ignoring the continuity correction were the common errors coupled with poor use of the appropriate significance test. Candidates need to have a simple algorithm at their fingertips to deal with tests of significance.

31. Most candidates were able to state the correct distribution, Bin(25, 0.25), and the hypotheses correctly. However, a sizeable minority were unable to identify the correct test statistic. The most common error was examining \( P(X = 10) \) instead of \( P(X > 10) \).

32. In part (a) many candidates struggled to explain the concept of a critical region, although some gave a correct
definition as the range of values where the null hypothesis is rejected. Many correct solutions were seen for parts (b) and (c). However the weaker candidates were not able to translate the concept ‘at most 4 breakdowns’ to the correct inequality. In part (d), as with Q3, many candidates successfully performed the required hypothesis test using a probability method. Again, there was a sizeable number of candidates who incorrectly found \( P(X=1) \) and compared this probability with the significance level. Again, a minority of candidates decided to approach this question using the critical region strategy. Marks were lost if candidates did not give evidence of their chosen critical region.

33. Candidates were able to express two conditions for a Poisson distribution in context with vehicles passing by a particular point on the road. Many candidates then answered part (b) and (c) correctly. In part (d) a majority of candidates was able to give a full solution by either using a probability or critical region approach to their hypothesis test.

34. Many candidates found this question difficult. A few candidates failed to look for the two tails in part (a) and, of those that did, many chose any value that was less than 2.5% rather than the closest value. Many identified the correct probability for the upper region, but then failed to interpret this as a correct critical region. Marks were lost by those who failed to show which values they had extracted from the tables to obtain their results. Nearly all of those who achieved full marks in part (a) answered part (b) correctly.

In part (c) weaker candidates had difficulty in stating hypotheses correctly and then attempted to use a Poisson distribution with a parameter obtained from dividing 74 by 6. However, the best candidates realised that a normal approximation was appropriate, with the most common error being an incorrect application of the continuity correction. Most solutions were placed in context.

35. Many good candidates lost marks carelessly by failing to show detailed working, even if they arrived at the correct critical region, and the statement of the region was often missing or stated as a probability. Some centres had candidates who were trying to follow the book method too closely and did not demonstrate a clear understanding of the concept of significance.

36. No Report available for this question.