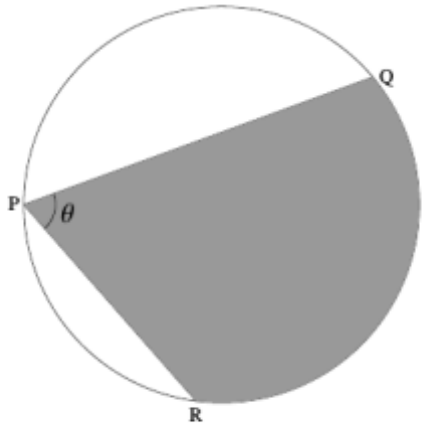
Using simple special cases in problem-solving

Another useful method, at least for checking answers, is to look at simple special cases of the problem, or simplified versions of the problem. We saw some of that when we talked about how to check your answers in FP1 and M2.
For example, in M2 problems involving a slope with angle θ you can check whether you have cos and sin in the right places in your equations by thinking about the case θ=0.
With some of the multiple-choice questions in MAT, that sort of argument is enough to give the complete answer.
Try these. Please write in your books, not just your multiple-choice answer, but the argument you used to reach the answer.
For at least one of the problems, a symmetry argument like those in the last worksheet is just as good.

1. MAT 2011, B. A rectangle has perimeter P and area A. The values P and A must satisfy: (a) $P^3 > A$, (b) $A^2 > 2P + 1$, (c) $P^2 \geq 16A$, (d) $PA \geq A+P$.

2. MAT 2012, J

J. If two chords QP and RP on a circle of radius 1 meet in an angle θ at P, for example as drawn in the diagram below,



then the largest possible area of the shaded region RPQ is

(a) $\theta\left(1+\cos\left(\frac{\theta}{2}\right)\right)$; (b) $\theta+\sin\theta$; (c) $\frac{\pi}{2}(1-\cos\theta)$; (d) θ.

STEP 1, 1999, Question 1

How many integers greater than or equal to zero and less than a million are not divisible by 2 or 5? What is the average value of these integers?

How many integers greater than or equal to zero and less than 4179 are not divisible by 3 or 7? What is the average value of these integers?

3. MAT 2010, B

B. The sum of the first $2n$ terms of

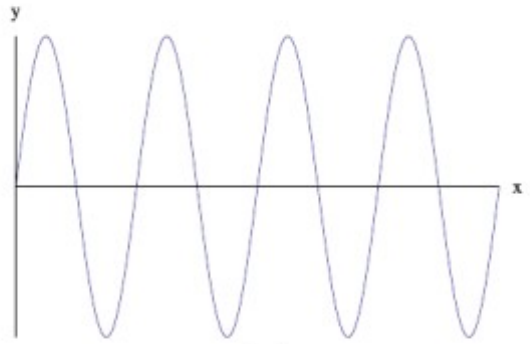
$$1, 1, 2, \frac{1}{2}, 4, \frac{1}{4}, 8, \frac{1}{8}, 16, \frac{1}{16}, \ldots$$

is

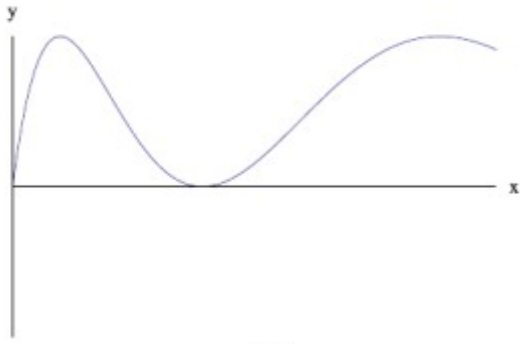
(a) $2^n + 1 - 2^{1-n}$, (b) $2^n + 2^{-n}$, (c) $2^{2n} - 2^{3-2n}$, (d) $\dfrac{2^n - 2^{-n}}{3}$.

4. MAT 2010, D

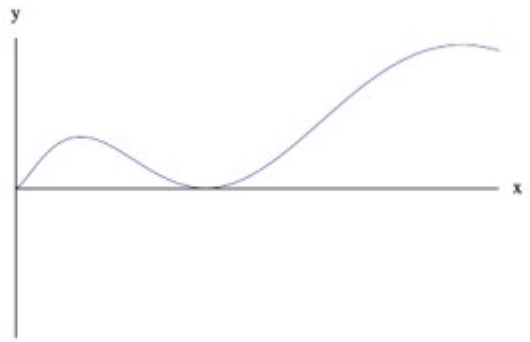
D. The graph of $y = \sin^2\sqrt{x}$ is drawn in



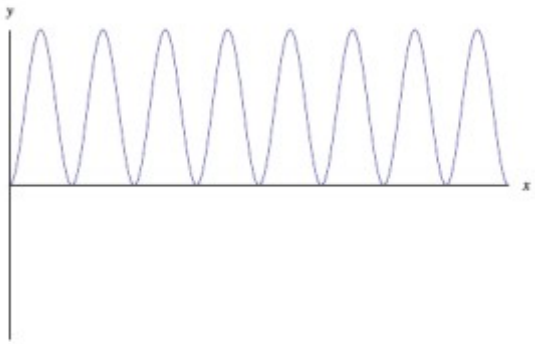
(a)



(b)



(c)



(d)

5. MAT, 2009, D

D. The smallest positive integer n such that

$$1 - 2 + 3 - 4 + 5 - 6 + \cdots + (-1)^{n+1} n \geqslant 100,$$

is

(a) 99, (b) 101, (c) 199, (d) 300.

6. (not a MAT question, but same sort of thing)
Without using a calculator, find which is bigger of 99^{100} and 100^{99}.