C3 resit paper, 9-10 February 2016

1.

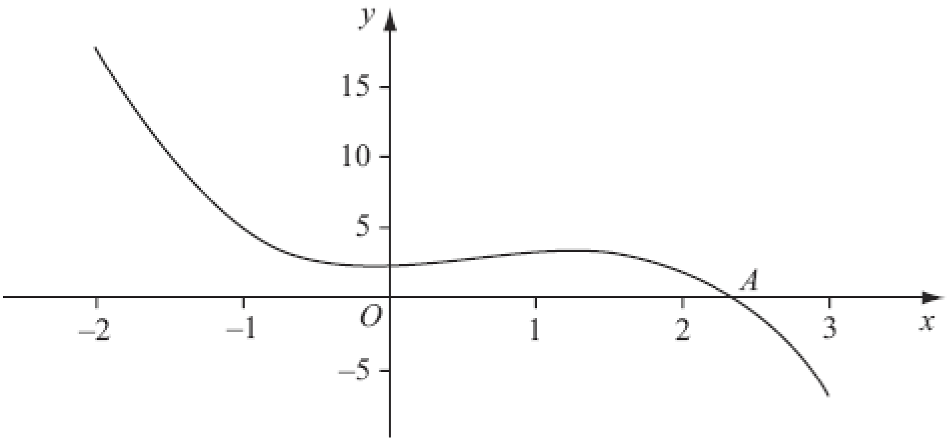

Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(*a*) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.

(3)

(*b*) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)

June 2009

2. $$\mathrm{f}(x) = x^3 + 3x^2 + 4x - 12$$

(*a*) Show that the equation $\mathrm{f}(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \quad x \neq -3.$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2.

(*b*) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \quad n \geq 0,$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of $\mathrm{f}(x) = 0$ is α.

(*c*) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

June 2012

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3. $$\mathrm{f}(x) = 2\sin(x^2) + x - 2, \qquad 0 \leq x < 2\pi.$$

(*a*) Show that $\mathrm{f}(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$.

(2)

The equation $\mathrm{f}(x) = 0$ can be written as $x = [\arcsin(1 - 0.5x)]^{\frac{1}{2}}$.

(*b*) Use the iterative formula

$$x_{n+1} = [\arcsin(1 - 0.5x_n)]^{\frac{1}{2}}, \qquad x_0 = 0.8,$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(*c*) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

(3)

June 2011

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4. $$f(x) = \ln(x + 2) - x + 1, \qquad x > -2, x \in \mathbb{R}.$$

(*a*) Show that there is a root of f(x) = 0 in the interval $2 < x < 3$.

(2)

(*b*) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2.5,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(*c*) Show that $x = 2.505$ is a root of f(x) = 0 correct to 3 decimal places.

(2)

January 2008

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5. The function f is defined by

$$f: x \mapsto \frac{2(x-1)}{x^2 - 2x - 3} - \frac{1}{x-3}, \quad x > 3.$$

(*a*) Show that $f(x) = \frac{1}{x+1}$, $x > 3$.

(4)

(*b*) Find the range of f.

(2)

(*c*) Find $f^{-1}(x)$. State the domain of this inverse function.

(3)

The function g is defined by

$$g: x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(*d*) Solve $fg(x) = \frac{1}{8}$.

(3)

June 2008

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6. The functions f and g are defined by

$$\text{f} : x \mapsto \ln(2x - 1), \qquad x \in \mathbb{R}, \quad x > \tfrac{1}{2},$$

$$\text{g} : x \mapsto \frac{2}{x-3}, \qquad x \in \mathbb{R}, \quad x \neq 3.$$

(*a*) Find the exact value of fg(4).

(2)

(*b*) Find the inverse function $\text{f}^{-1}(x)$, stating its domain.

(4)

(*c*) Sketch the graph of $y = |\text{g}(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y-axis.

(3)

(*d*) Find the exact values of x for which $\left|\dfrac{2}{x-3}\right| = 3$.

(3)

June 2007

7.

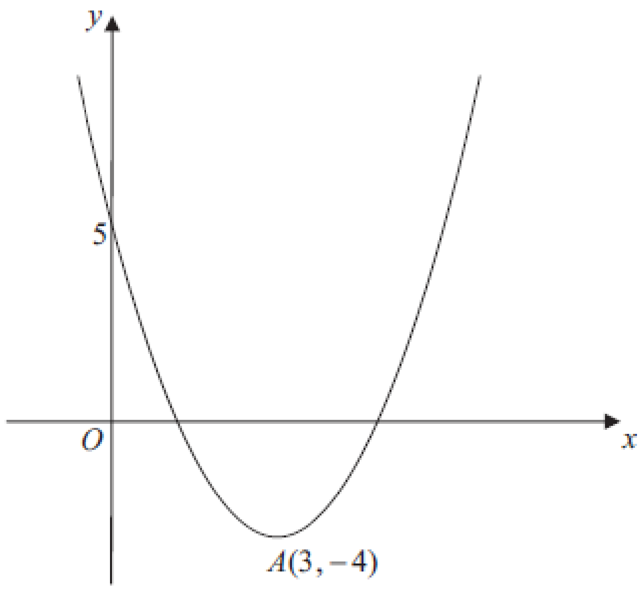

Figure 2

Figure 2 shows a sketch of the curve with the equation $y = \text{f}(x)$, $x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point (0, 5).

(*a*) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |\text{f}(x)|$,

(ii) $y = 2\text{f}(\tfrac{1}{2}x)$.

(4)

(*b*) Sketch the curve with equation $y = \text{f}(|x|)$.

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

(3)

The curve with equation $y = \text{f}(x)$ is a translation of the curve with equation $y = x^2$.

(*c*) Find f(x).

(2)

(*d*) Explain why the function f does not have an inverse.

(1)

June 2010

8. (*a*) Differentiate with respect to x,

(i) $x^{\frac{1}{2}} \ln (3x)$,

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(*b*) Given that $x = 3 \tan 2y$ find $\dfrac{dy}{dx}$ in terms of x.

(5)

June 2012

TOTAL FOR PAPER: 75 MARKS

END (This is Edexcel's "Silver Two" paper)