FP1 May 2016 Mark scheme

Question Number	Scheme	Marks
1.	Determinant of **A** = $(1-k)(1+k) - k^2 = 0$	M1
	$1-k+k-k^2-k^2 (=0)$ $1-2k^2(=0)$ So $k = \dfrac{\pm\sqrt{2}}{2}$	A1 A1 (3)
		3 marks

Notes

M1: for attempting $ad - bc = 0$ with $'=0'$ seen or **used** later in the solution.

A1: Correct (unsimplified) expression on LHS or correct equation after brackets expanded.

A1: Accept $\pm\dfrac{\sqrt{2}}{2}$, $\pm\dfrac{1}{\sqrt{2}}$, $\pm\sqrt{\dfrac{1}{2}}$, $\pm\sqrt{0.5}$. Must have $\pm$ for mark.

Question Number	Scheme	Marks
2.	(a) $f'(x) = \frac{9}{2}x^{\frac{1}{2}} + \frac{25}{2}x^{-\frac{3}{2}}$	M1 A1
		(2)
	(b) f(12.5)= 0.5115… (at least 0.51…) and $f'(12.5) = 16.1927...$ (at least 16… seen)	B1, B1
	$x_1 = 12.5 - \frac{f(12.5)}{f'(12.5)} = 12.5 - \frac{0.5115}{16.1927...} = 12.468$	M1 A1
		(4)
		6 marks

Notes

(a) M1: for attempting differentiation i.e. decrease a power by 1
A1 Accept equivalent expression i.e. condone equivalent fractions.

(b) B1: One correct, must be explicitly seen if final answer incorrect, may be implied by correct final answer.

B1: Both correct; must be explicitly seen if final answer incorrect, may be implied by correct final answer.

M1:for attempting Newton- Raphson with their values for f(12.5) and $f'(12.5)$

A1: cao correct to 3dp

Newton Raphson used more than once – isw.

Question Number	Scheme	Marks
3.	(a) $\sum_{r=1}^{3n} r^2 = \frac{1}{6}3n(3n+1)(6n+1)$ or $\sum_{r=1}^{3n} r^2 = \frac{1}{2}n(3n+1)(6n+1)$ or equivalent	B1 (1)
	(b) See $\sum_{r=1}^{2n} r^2 = \frac{1}{3}n(2n+1)(4n+1)$ or equivalent	B1
	Attempt to use $\sum_{r=1}^{3n} r^2 - \sum_{r=1}^{2n} r^2 = \frac{n}{6}\{3(3n+1)(6n+1) - 2(2n+1)(4n+1)\}$	M1
	$= \frac{n}{6}\{(54n^2 + 27n + 3) - (16n^2 + 12n + 2)\}$	dM1
	$= \frac{n}{6}\{(38n^2 + 15n + 1)\}$ $(a = 38, b = 15, c = 1)$	A1
		(4)
		5 marks

Notes

(a) B1: Either right hand side or exact equivalent - isw if expanded

(b) B1: States or uses $\sum_{r=1}^{2n} r^2 = \frac{1}{3}n(2n+1)(4n+1)$

M1: Subtracts their sum to $2n$ or $2n - 1$ **and** attempts to factorise by $\frac{n}{6}$ seen anywhere.

dM1: Expands two quadratics dependent on first M1

A1: cao

Question Number	Scheme	Marks
4.	(a) $z = \frac{4(1-\text{i})}{(1+\text{i})(1-\text{i})}$	M1
	$z = 2(1-\text{i})$ or 2 – 2i or exact equivalent.	A1 (2)
	(b) $z^2 = (2-2\text{i})(2-2\text{i}) = 4-8\text{i}+4\text{i}^2$	M1
	$= -8\text{i}$	A1 cao (2)
	(c) If z is a root so is z^* So $(x-2+2\text{i})(x-2-2\text{i})$ (or $x^2 - 2\text{Re}(z).x + \lvert z\rvert^2$)	M1
	So $(x-2+2\text{i})(x-2-2\text{i}) = 0$ (or $x^2 - 2\text{Re}(z).x + \lvert z\rvert^2 = 0$) and so $p = q =$	M1
	Equation is $x^2 - 4x + 8 (= 0)$ or $p = -4$ and $q = 8$	A1 (3) (7 marks)
ALT 1	(c) Substitutes $z = 2 - 2\text{i}$ and $z^2 = -8\text{i}$ into quadratic and equates real and imaginary parts to obtain $2p + q = 0$ and $-2p - 8 = 0$ Attempts to solve simultaneous equations to obtain $p = -4$ and $q = 8$	 M1 M1A1
ALT 2	(c) Attempts to obtain $p = -$ sum of roots Attempts product of roots to obtain $q=$	 M1 M1
	Equation is $x^2 - 4x + 8 (= 0)$ or $p = -4$ and $q = 8$	A1
ALT 3	(c) $x - 2 = \pm 2\text{i}$ either sign acceptable	M1
	$(x-2)^2 = -4 \Rightarrow x^2 - 4x + 4 = -4$ i.e square and attempt to expand to give 3-term quadratic	M1
	Equation is $x^2 - 4x + 8 (= 0)$ or $p = -4$ and $q = 8$	A1

Notes

(a) M1: Multiplies numerator and denominator by 1 – i or by -1 + i
A1: cao
(b) M1: Squares their z, or the given $z = \frac{4}{1+\text{i}}$, to produce at least 3 terms which can be implied by the correct answer.
A1: -8i or $0 - 8\text{i}$ only
(c) M1: Uses their z and z^* in $(x-z)(x-z^*)$
M1: Multiplies two factors and obtains $p =$ or $q =$
A1: Both correct required – can be implied by $x^2 - 4x + 8$
ALT 1
(c) M1: Substitutes their z and their z^2 into the quadratic and equates real and imaginary parts to obtain two equations in p and q
M1: Attempts to solve for one unknown to obtain $p =$ or $q =$
A1: Both correct required – can be implied by $x^2 - 4x + 8 (= 0)$

Question Number	Scheme	Marks
5.	(a) Gradient $=\dfrac{2ap-2aq}{ap^2-aq^2}$ seen	B1
	$\left(\dfrac{2ap-2aq}{ap^2-aq^2}=\dfrac{2a(p-q)}{a(p-q)(p+q)}\right)=\dfrac{2}{p+q}$ or seen in an equation	B1
	uses $y-y_1=m(x-x_1)$ to give $(y-2aq)="m"(x-aq^2)$ or equivalent with p or uses $y = mx + c$ to give $y="m"x+c$ and substitute a point to find c or uses $\dfrac{y-y_1}{y_2-y_1}=\dfrac{x-x_1}{x_2-x_1}$ to give $\dfrac{(y-2aq)}{2ap-2aq}=\dfrac{(x-aq^2)}{ap^2-aq^2}$ or equivalent with p	M1
	so $(y-2aq)=\dfrac{2}{p+q}(x-aq^2)$ or $(y-2ap)=\dfrac{2}{p+q}(x-ap^2)$ or $y=\dfrac{2}{p+q}x+\dfrac{2apq}{p+q}$ or $\dfrac{(y-2aq)}{2a}=\dfrac{(x-aq^2)}{a(p+q)}$	A1
	See $2aq^2$ or $2ap^2$ term appear and disappear to give $y(p+q)=2x+2apq$ *	A1 cso (5)
	(b) Substitute (a , 0) into line equation, to give $0=2a+2apq$ so $pq=-1$	B1 (1)
	(c) $y=2a^{\frac{1}{2}}x^{\frac{1}{2}}\Rightarrow\dfrac{dy}{dx}=a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $y^2=4ax\Rightarrow 2y\dfrac{dy}{dx}=4a$ or $\dfrac{dy}{dx}=\dfrac{dy}{dp}\times\dfrac{dp}{dx}\Rightarrow\dfrac{dy}{dx}=2a\times\dfrac{1}{2ap}$	M1
	So at P tangent gradient = $\dfrac{1}{p}$	A1 (2)
	(d) At Q tangent gradient = $\dfrac{1}{q}$	B1
	$\dfrac{1}{p}\times\dfrac{1}{q}=\dfrac{1}{pq}=\dfrac{1}{-1}=-1$ with at least one intermediate step, the tangents are **perpendicular** or at **right angles**	B1cso (2) (10 marks)

Notes

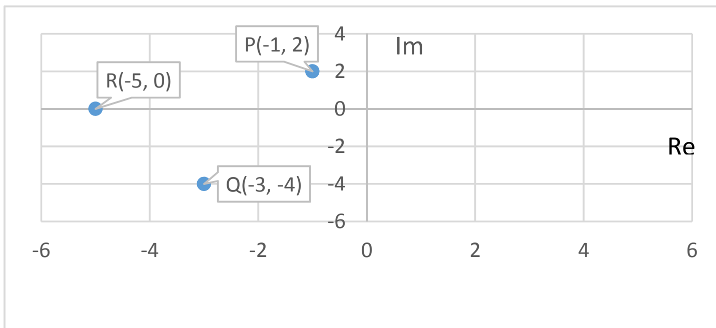
(a) B1: Correct statement for gradient (isw) B1: $\dfrac{2}{p+q}$ - can be seen later in the solution.

M1: use of a correct formula for a line equation through P or Q with their gradient.
Must be finding a chord, not a tangent or a normal.
A1: for a correct line equation with simplified gradient in any equivalent form
A1: cso (as given answer)
(b) B1: For using (a, 0) to show that pq = -1
(c) M1: Use calculus to find an expression for dy/dx and substitute coordinates of P.
They may use chord gradient and let p tend to q.
(d) B1: 1/q seen B1: $\dfrac{1}{p}\times\dfrac{1}{q}=-1$ or $\dfrac{1}{p}=-\dfrac{1}{1/q}$ or $\dfrac{1}{q}=-\dfrac{1}{1/p}$ and at least words in bold with no errors seen.

Question Number	Scheme	Marks
6.	(a) **Rotation** ,135 degrees or $\dfrac{3\pi}{4}$ radians (anticlockwise) about *O* or 225 degrees or $\dfrac{5\pi}{4}$ clockwise about *O*.	M1, A1 (2)
	(b) $\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 6\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$	
	$-p-q=12$ and $p-q=6$ or equivalent	M1A1
	$p = -3$ and $q = -9$ or $\begin{pmatrix} -3 \\ -9 \end{pmatrix}$	B1 cso (3)
	ALT Uses Inverse matrix $\mathbf{P}^{-1}$ with vector $= \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 6\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$	M1A1
	$p = -3$ and $q = -9$ or $\begin{pmatrix} -3 \\ -9 \end{pmatrix}$	B1 cso (3)
	(c) $\mathbf{Q} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1 (1)
	Accept **T** if used instead of **R**	
	(d) $\mathbf{R} = "\mathbf{Q}"\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1 A1 A1 (3)
	(e) $\mathbf{R}^{-1} = \dfrac{1}{-1}\begin{pmatrix} -\frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \\ +\frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	
	$= \mathbf{R}$ (so matrix is self inverse and so transformation is self inverse)	B1 **(1)** **10 marks**
	ALT 1 (e) $\mathbf{RR} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (so **R** is self inverse and so transformation is self inverse)	B1
	ALT 2 (e) Matrix represents a reflection (so is self inverse)	B1

Notes

(a) M1: Rotation only A1: 135 degrees about *O*
SC: 135 degrees about *O* only award M1A0.
(b) M1: Multiplies matrices in correct order to obtain two equations in *p* and *q*.
A1: Two correct equations
B1 cso: *p* and *q* both correct, may be in vector form. No errors seen in solution.
ALT (b) M1: Attempt to find Inverse Matrix and pre-multiply A1: Correct Inverse Matrix used
B1 cso: *p* and *q* both correct, may be in vector form. No errors seen in solution.
(d) M1: Sets matrix product correct way round and obtains one correct term for their **Q**
A1: Two correct terms from a correct **Q.** **Q** incorrect award A0 here. A1: Completely correct matrix
(e) B1: Calculates $\mathbf{R}^{-1}$ and indicates that $\mathbf{R}^{-1} = \mathbf{R}$ or calculates $\mathbf{R}^2$ and indicates that $\mathbf{R}^2 = \mathbf{I}$ or states that **R** represents a reflection.

Question Number	Scheme	Marks
7.	(a) $z^2 = (a+2\text{i})(a+2\text{i}) = (a^2-4)+4\text{i}a$ So $z^2+2z = (a^2-4+2a)+\text{i}(4a+4)$ or $x=(a^2+2a-4)$ and $y=4a+4$	M1 M1 A1 A1 (4)
	(b) and so $4a+4=0 \to a=-1$	B1 (1)
	ALT (b) Substitute $a=-1$ and show that $y=0$	B1
	(c) $\lvert z\rvert = \sqrt{5}$ or awrt 2.24 $\arctan,(-2) = 2.03$	B1 M1, A1 cao (3)
	(d) 	M1 A1 B1ft (3)
	(e) OP and QR are parallel , and QR is twice the length of OP Or Enlargement with Scale Factor 2 (centre O), followed by translation $\begin{pmatrix}-3\\-4\end{pmatrix}$ Or Enlargement with Scale Factor 2, centre (3,4) or centre 3 + 4i $\overrightarrow{QR} = 2\overrightarrow{OP}$ with clear indication of vectors award B1B1, without vectors award B0B1	B1, B1 (2) **13 marks**

Notes:

(a) M1: Squares z to produce at least 3 terms which can be implied by the correct answer.
M1: Adds $2z$ to their z^2
A1: Correct x A1 Correct y accept $4a\text{i}+4\text{i}$
(b) B1: Completely accurate cao
(c) B1: √5 or 2.24 or awrt 2.24
M1 for using tan or arctan
A1 cao 2.03
(d) M1: **Either** their OP in the correct quadrant labelled P or z or their $-1+2\text{i}$ or their (-1,2) or axes labelled **or** their OQ in the correct quadrant labelled Q or z^2 or their $-3-4\text{i}$ or their (-3, -4) or axes labelled
A1: Both OP and OQ correct i.e. in the 2nd and 3rd quadrants respectively.
B1ft: $OR: z^2+2z(=-5)$ **on real axis** to left of the origin.
Accept points or lines. Arrows not required. Axes need not be labelled Re and Im.
Treat correct quadrant (or on axis) as important aspect for accuracy, lengths of lines if present can be accepted as correct.

Question Number	Scheme	Marks
8.	(i) If $n=1$, $\sum_{r=1}^{n}\frac{2r+1}{r^2(r+1)^2}=\frac{3}{4}$ **and** $1-\frac{1}{(n+1)^2}=\frac{3}{4}$**,** so true for $n = 1$.	B1
	Assume result true for $n = k$ **and** consider $\sum_{r=1}^{k+1}\frac{2r+1}{r^2(r+1)^2}=1-\frac{1}{(k+1)^2}+\frac{2(k+1)+1}{(k+1)^2(k+2)^2}$	M1
	$=1-\left(\frac{(k+2)^2}{(k+1)^2(k+2)^2}-\frac{2(k+1)+1}{(k+1)^2(k+2)^2}\right)=1-\left(\frac{(k^2+2k+1)}{(k+1)^2(k+2)^2}\right)$	A1
	$=1-\left(\frac{(k+1)^2}{(k+1)^2(k+2)^2}\right)=1-\left(\frac{1}{(k+1+1)^2}\right)$	M1
	True for ***n* = *k* + 1** if **true** for ***n* = *k***, (and **true** for ***n* = 1**) so **true** by induction for all $\boldsymbol{n\in Z^+}$	A1cso (5)
	(ii) $n = 1$: $u_1=5\times\left(\frac{1}{3}\right)^1+\frac{4}{3}=3$ so expression for u_n true for $n = 1$	B1
	Assume result true for $n = k$ **and** consider $u_{k+1}=\frac{1}{3}(5\times\left(\frac{1}{3}\right)^k+\frac{4}{3})+\frac{8}{9}$	M1
	Obtain $u_{k+1}=5\times\left(\frac{1}{3}\right)^{k+1}+\frac{4}{9}+\frac{8}{9}$	A1
	$5\times\left(\frac{1}{3}\right)^{k+1}+\frac{4}{3}$ and deduce that result is true for $n = k + 1$	dM1
	True for ***n* = *k* + 1** if **true** for ***n* = *k***, (and **true** for ***n* = 1**) so **true** by induction for all $\boldsymbol{n\in Z^+}$	A1 cso (5) **10 marks**

Notes:

(i) B1: Checks $n = 1$ on both sides **and** states true for $n = 1$ seen anywhere

M1: (Assumes true for) $n = k$ **and** adds $(k+1)^{\text{th}}$ term to sum of k terms

A1: $1-\left(\frac{(k^2+2k+1)}{(k+1)^2(k+2)^2}\right)$ seen (linked to 2nd M)

M1: $(k+1)^2(k+2)^2$ attempted as common denominator of two fractions.

A1cso: Makes correct complete induction statement including at least statements in bold. Accept $n\geq 1$ or $n=1,2,3...$ or all positive Integers or all n. Statement true for $n = 1$ here could contribute to B1 mark earlier.

(ii) B1: Checks $n = 1$ in u_n and states true for $n = 1$ seen anywhere.

M1: (Assumes result for) $n = k$ **and** substitutes u_k into correct expression for u_{k+1}

A1: $\frac{4}{9}+\frac{8}{9}$ or $\frac{1}{3}.\frac{4}{3}+\frac{8}{9}$ seen

dM1: Obtains $5\times\left(\frac{1}{3}\right)^{k+1}+\frac{4}{3}$ and statement true for $n = k + 1$ or equivalent seen anywhere dependent on previous M.

A1cso: Makes correct complete induction statement including at least statements in bold. Accept $n\geq 1$ or $n=1,2,3...$ or all positive Integers or all n. Statement true for $n = 1$ here could contribute to B1 mark earlier.

Question Number	Scheme	Marks
9 (a)	$y=\frac{25}{x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-25x^{-2}$,	B1
	or $y+x\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{x}$ **or** $\dot{x}=5,\ \dot{y}=-\frac{5}{t^2}$ so $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{t^2}$	
	and at P $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{p^2}$ so gradient of normal is p^2	M1 A1
	Either $y-\frac{5}{p}=p^2(x-5p)$ **or** $y=p^2x+k$ and use $x=5p,\ y=\frac{5}{p}$	M1
	$\Rightarrow\quad y-p^2x=\frac{5}{p}-5p^3$ (*)	A1cso (5)
(b)	At the point A: $y+p^2y=\frac{5}{p}-5p^3$ or $-x-p^2x=\frac{5}{p}-5p^3$	
	$y(1+p^2)=\frac{5}{p}(1-p^4)$ or $-x(1+p^2)=\frac{5}{p}(1-p^4)$	M1
	$y=\frac{\frac{5}{p}(1-p^2)(1+p^2)}{(1+p^2)}=\frac{5}{p}(1-p^2)=\frac{5}{p}-5p$ or $x=\frac{-\frac{5}{p}(1-p^2)(1+p^2)}{(1+p^2)}=\frac{-5}{p}(1-p^2)=\frac{-5}{p}+5p$ *	M1
	so $x=-\frac{5}{p}(1-p^2)=-\frac{5}{p}+5p$ and $y=\frac{5}{p}-5p$ *	A1cso (3)
(c)	M has coordinates $\left(-\frac{5}{2p}+5p,\ \frac{5}{p}-\frac{5p}{2}\right)$ o.e.	B1
	So when $y=0$, $\frac{5}{p}-\frac{5p}{2}=0$ and $p=\sqrt{2}$ so M has x coordinate $\frac{15}{4}\sqrt{2}$ o.e.	M1 A1 (3)
		11 marks

Notes

(a) B1: Any correct expression for gradient of tangent
M1: Substitutes values into **derived expression using calculus** to give gradient of normal at P
A1: cao. Can be implied by use in equation of a straight line
M1: Use of formula for the equation of a straight line with their changed gradient
A1: cso
(b) M1: Replaces x by $-y$ or y by $-x$
M1: Factorises $(1-p^4)$ to simplify answer in first variable
A1 cso: Obtains both x and y
ALT (b) Accept Verification.
M1: Substitutes the coordinates of A into the equation of the normal
M1: Substitutes the coordinates of A into both the normal and $y = -x$.
A1 cso: No errors seen
(c) B1: Correct x- coordinate of midpoint (may be implied) and correct y coordinate, accept equivalent forms
M1: Puts their $y = 0$ and finds value for p to use in $x =$
A1: $+\frac{15}{4}\sqrt{2}$ or equivalent only