Step 2 of proof that $2304 \mid 7^{2n} - 48n - 1$ for all n

To prove: $2304 \mid 7^{2k+2} - 48(k+1) - 1$

True for n=k => $2304 \mid 7^{2k} - 48k - 1$

$\Rightarrow$ $2304 \mid 7^{2k+2} - 49\times48k - 49$ $(\times7^2)$

$\Rightarrow$ $2304 \mid 7^{2k+2} - 2352k - 48 - 1$

$\Rightarrow$ $2304 \mid 7^{2k+2} - 48k - 48 - 1$ $(+ 2304k)$

$\Rightarrow$ $2304 \mid 7^{2k+2} - 48(k+1) - 1$ □

Please copy Bolaji's idea and write a description (like ($\times7^2$) or (+ 2304k)) of what you've done to the right-hand side at each stage. This helps the marker understand you, and helps you avoid mistakes.

Simultaneous equations: A= $\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$ maps $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$. You want to find x and y, and you already know A^{-1} = $\frac{1}{14}\begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix}$

You can do this by writing $\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$

and calculating that out to get two simultaneous equations in x and y.

It is slightly quicker to use: A $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$ => $\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$

So $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix}\begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$ – you get the answer straight from the matrix calculation, without then having to do simultaneous equations.

Square roots of complex numbers. Find the square root of -3+4i

<u>Method 1:</u> **Pol**(-3, 4) => (r=5, θ=2.2142974…)

Square-rooting square-roots the modulus, but halves the argument

So modulus of square root = $\pm\sqrt{5}$

For argument of square root, do Alpha>Y= to get your θ value stored as Ans; then do Ans/2 =, so you get $\theta/2$ stored as Ans

Then convert back to ordinary x+iy form (called Cartesian) by doing **Rec**($\sqrt{5}$,Ans)

Rec($\sqrt{5}$,Ans) => X=1, Y=2, so square roots are $\pm(1+2i)$

Rec is the reverse of **Pol**.

<u>Method 2:</u> Call square root a+bi

Real part of $(a+bi)^2 = a^2 - b^2$ = real part of $-3+4i = -3$

$|a+bi|^2 = a^2 + b^2 = |-3+4i| = 5$

Now we have an easy set of simultaneous equations for a^2 and b^2. Solve them: $a^2= 1$, $b^2=4$.

$a = \pm1$, $b= \pm2$. Check which of the $\pm$ values apply. Answer: square roots are $\pm(1+2i)$.

Find λ if λ is real, z= 5 + i√3, and arg (z + λ) = π/3

Best do problems like this with a diagram. Draw the half-line of all the points which have argument = π/3.

Mark the point z

Draw the line of all the points you get as z+λ for different real values of λ.

The λ you want is where that line crosses the half-line of points with argument = π/3.

Work out λ from the diagram. In this case, since the z+λ we want is √3 units above the real axis, it must be 1 unit to the right of the imaginary axis (from the measurements of the equilateral triangle shown). So λ = -4.

Better to do it like this than by calculations of tan - because it's simpler - because you can see what you're doing - because if the problem said arg (z + λ) = π/2 you'd have trouble working with tan because tan(π/2) gives you "Math Error".