Things done differently 2017-8

City of London Academy Southwark

1 Square roots of complex numbers

I've recommended two alternative methods to the "official" one, which is to find equations for $a^2 - b^2$ and for $2ab$ where a and b are the real and imaginary parts of the square root.

Namely: (1) use the calculator to find the argument of the number to be square-rooted;

calculate Ans÷2

calculate $\sqrt{\text{mod}}\angle$Ans

convert back to cartesian form

and check!

and write the negative of that answer as the other square root

(2) put $a^2 - b^2 = \text{Re}(z)$ and $a^2 + b^2 = |z|$, solve for a^2 and b^2, check the signs for a and b.

See https://mathsmartinthomas.wordpress.com/2017/10/08/finding-square-roots-of-complex-numbers/

2 Proof by induction

I've recommended almost exactly the words used by Edexcel for the "conclusion", and checked by writing to Edexcel that they will be accepted.

For the "basis" step, which I call just "Step 1" to minimise jargon, I've suggested the following wording:

Prove - true for $n = 1$

The following is just as good, though a bit longer

Prove that the claim is true for $n = 1$

The following is even shorter, and will do.

Prove for $n = 1$

Some students sometimes write

"Prove that $n = 1$"

I can guess they mean: prove that *the claim is true for* $n = 1$. But what you've written isn't a good abbreviation. "Prove that $n = 1$" is not a good wording.

For Step 2 (the "induction" step), I've suggested

Prove - true for $n = k \Rightarrow$ true for $n = k + 1$

The following is just as good, and only a little longer

Prove - if true for $n = k$, then true for $n = k + 1$

The following is strictly speaking not correct, since $\Rightarrow$ means "if... then". "It is raining $\Rightarrow$ the ground gets wet" means "if it is raining, then the ground gets wet". So the "if" in the following is not necessary. I don't think you'd lose marks for it, though.

Prove - if true for $n = k \Rightarrow$ true for $n = k + 1$

The textbook suggests something like the next one.

Assume true for $n = k$. Prove true for $n = k + 1$

Real mathematicians never write it that way. I don't like it. How can you "assume" the claim is true for $n = k$? You can't. And you don't need to. You don't have to assume it's actually raining to prove that *if* it rains, *then* the ground will get wet. However, since the textbook likes it, the Edexcel markers will like it.

The following is not correct at all, though I can guess what you *mean* is "Prove that if *the claim is true for n = k* then *the claim is true for n = k + 1*" (which is correct, but long)

"Prove that if $n = k$ then $n = k + 1$"

For the conclusion I have suggested:

The claim is true for $n = 1$. For all k, we have shown that *if* it is true for $n = k$ *then* it is true for $n = k + 1$ So, by mathematical induction, it is true for all $n \in \mathbb{N}$ $\square$

I've suggested they stick quite closely to this.

For proofs by induction by divisibility, I've explained that Edexcel's method of always working out $f(k+1)-f(k)$ *sometimes* makes the proof no longer, but never expedites it, and often is a completely useless step which makes the proof longer. So I suggest the following sort of setting-out.

Notice at the start of Step 2 (the induction step) the writing-out of the "to prove" proposition in a form that makes it easier to see when "you've arrived" in the proof.

For the first move in Step 2, I suggest the students first look to see if they can get where they want by *adding* (a multiple of the divisor) to the RHS. Then, if they can't do that (as in the example below), *multiply* the RHS by a suitable amount to get it near what they want.

Example:

To prove: $19 \mid 3^{3n-2} + 2^{3n+1}$

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1. Prove - true for $n = 1$

$$3^1 + 2^4 = 19 \qquad \square$$

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2. Prove - true for $n = k \Rightarrow$ true for $n = k+1$

To prove: $19 \mid 3^{3k+1} + 2^{3k+4}$, or $19 \mid 27 \times 3^{3k-2} + 8 \times 2^{3k+1}$

True for $n = k \Rightarrow 19 \mid 3^{3k-2} + 2^{3k+1}$

$\ldots \Rightarrow 19 \mid 27 \times 3^{3k-2} + 27 \times 2^{3k+1}$ (RHS $\times 27$)

$\ldots \Rightarrow 19 \mid 27 \times 3^{3k-2} + 8 \times 2^{3k+1}$ (RHS $-19 \times 2^{3k+1}$) $\square$

3 Making new equations whose roots are a+1, b+1, c+1 or whatever from an equation with roots a, b, c

This is not really something done differently from the book, but I've emphasised more than the book does the need to choose between two methods. Either - in an example like the above - you put $y = x + 1$ (if the original equation is written in terms of x), then plug $y - 1 = x$ into the x-equation to get the new equation you want (written in terms of y). Or you use Vieta formulas to find the sum, sum of products-in-pairs, and product of roots of the new equation.

You *have to* use the second method if the new equation is to have roots, say, a^2, b^2, c^2. But the first method is *much* quicker, and less vulnerable to slips in working, if the new equation is to have roots, say, $a+1, b+1, c+1$ (linear functions of the original roots) or $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$. (Or, in general, a Möbius transformation of the original roots).

4 FP3 vectors

See https://mathsmartinthomas.wordpress.com/2015/02/27/dont-do-as-the-book-says-fp3-vectors/. The main thing there is finding the distance from a point $\mathbf{s}$ to a line $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ from the area of the parallelogram made by $\mathbf{s} - \mathbf{a}$ and $\mathbf{b}$, rather than from calculus. I've also simplified the methods of finding the line of intersection of two planes, and for finding the distance between two parallel lines.

Also, I suggest to the students that they find distance from a point to a plane, and distance from a line parallel to a plane to the plane, by adapting the method they already know for the distance between two parallel planes, rather than (as the textbook and Solution Bank do) searching around for longer and more complicated methods (why?)

Distance between planes $\mathbf{r} \cdot \hat{\mathbf{n}} = p_1$ and $\mathbf{r} \cdot \hat{\mathbf{n}} = p_2$ is $|p_1 - p_2|$

Distance between point $\mathbf{a}$ and plane $\mathbf{r} \cdot \hat{\mathbf{n}} = p$ is the distance between the plane $\mathbf{r} \cdot \hat{\mathbf{n}} = p$ and a plane parallel to it containing $\mathbf{a}$, and so $|\mathbf{a} \cdot \hat{\mathbf{n}} - p|$

Distance between plane $\mathbf{r} \cdot \hat{\mathbf{n}} = p$ and a line $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ parallel to it = the distance between the point $\mathbf{a}$ and the plane.